



## [Module 7 Overview Document](#)

Table 1: Timeline of Tasks in the Module

|  |       |          |   |
|--|-------|----------|---|
| <b>Timeline of tasks in the Module</b> | Day 0 | Homework | 7.1 Engage in Introduction to the Sine Graph Desmos Activity  |
|  | Day 1 | 20 min   | 7.1 Discussion of Introduction to the Sine Function Desmos Activity<br>Optional: Extend the Discussion: Task Design |
|  |       | 35 min   | 7.2 Launching a Technology-Mediated Math Task   |
|  |       | 20 min   | 7.3 Noticing Student - Teacher Interactions   |
|  |       | Homework | 7.4 Monitoring Student Thinking: Introduction to the Sine Function  |
|  | Day 2 | 15 min   | 7.4 Discussion of Introduction to the Sine Function: Monitoring Student Thinking                                    |
|  |       | 20 min   | 7.5 Noticing Student Thinking about Amplitude   |
|  |       | 40 min   | 7.6 Noticing Student Thinking about Period  |
|  | Day 3 | 40 min   | 7.7 Designing a Sequence of Tasks (optional project)  |
|  |       |          |   |

## 7.7 Facilitation Notes

This project is an addition to Module 7 for courses that have a focus on teaching with technology. If you have been focused on task selection, adaptation, and/or design this is a great opportunity to continue to build those skills while also focusing on sequences of tasks (i.e., tasks that build on each other toward specific learning goals). If you are working with teachers who are working toward licensure in a state that requires edTPA, connecting this to the edTPA sequence of lessons is helpful.

Prior to introducing the project, to build an understanding of the difference between tasks that are intended to support students in learning a new idea, support students in further developing an idea, and applying their knowledge it is helpful to have teachers explore the activities in the search engine at [teacher.desmos.com](http://teacher.desmos.com). Each of these activities has been labeled as “introduction”, “development”, “application” or “practice”. Having teachers select a specific topic and compare and contrast tasks related to that topic but labeled differently. Then discuss what would be important to look for (or if you are creating your own task, to include) when selecting a task intended to a) introduce a new concept, b) further develop that concept, c) provide opportunities to practice, and d) to apply a concept.

After this discussion, introduce the project and provide time for research and brainstorming. It is helpful for teachers to work collaboratively to brainstorm ideas for their task, but we recommend having them build their tasks outside of class time. A



follow up assignment could be to engage with each other's tasks and provide feedback. In this way, everyone will have a nice selection of tasks to sequence for an instructional unit.

Having teachers justify their design choices by referencing prior readings is important. This could be aligned with revisiting the following:

- PTMT Algebra Module Chapter 2, Section 2: Evaluating Technology Tools and Section 3: Classifying, Adapting, and Using Tasks (pg. 25 - 32).
- Readings from Module 1
  - McCulloch, A. W., Lovett, J. N., Dick, L. K., & Cayton, C. (2021). Positioning each and every student as a mathematical explorer with technology. *Mathematics Teacher: Learning and Teaching PK-12: Special issue on Digital Equity and the Digital Divide*, 114(10), 738-749, 10.5951/MTLT.2021.0059
  - Dick, L. K., McCulloch, A. W., & Lovett, J. N. (2021). When students use technology tools, what are you noticing? *Mathematics Teacher: Learning and Teaching PK-12*, 114(4), 272 – 283.
- Readings from Module 6
  - Bailey, N. G., Yalman Ozen, D., Lovett, J. N., McCulloch, A. W., & Cayton, C. (2021). Parameters, sliders, marble slides, oh my! *Mathematics Teacher: Learning and Teaching PK-12*, 114(5), 386-394.
  - Lo, J. J., & White, N. (2020). Selecting GeoGebra applets for learning goals. *Mathematics Teacher: Learning and Teaching PK-12*, 113(2), 156–159.
  - Hunt, J., & Stein, M. K. (2020). Constructing goals for student learning through conversation. *Mathematics Teacher: Learning and Teaching PK-12*, 113(11), 904–909.
- From Module 7
  - Jackson, K. J., Shahan, E. C., Gibbons, L. K., & Cobb, P. A. (2012). Launching complex tasks. *Mathematics Teaching in the Middle School*, 18(1), 24 - 29.
  - NC<sup>2</sup>ML (2018, February). Launching a task: Providing opportunities for all students to learn. *Research-Practice Briefs*. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. Retrieved from [nc2ml.org/brief22](http://nc2ml.org/brief22).

## 7.6 Sample Responses

### Designing a Sequence of Tasks

#### Context

Ms. Fye's students have completed the Introduction to Sine Graphs task where they learned about amplitude, midline, and period of the sine function. In the next few days she plans to provide her students opportunities to further develop and practice their



understanding of amplitude, midline, and period, to apply this knowledge, and also to introduce them to phase shift.



### [Introduction to Sine Desmos Activity](#)

For this assignment you will design a next task in this sequence for Ms. Fye's class. It can be any of the following:

- a task that provides an opportunity for her students **to further develop** their understanding of amplitude, midline, and period related to sine functions and their graphs.
- a task that provides an opportunity for her students **to apply** their knowledge of amplitude, midline, and period to a real context through modeling.
- an investigative task intended **to introduce** phase shift to go along with amplitude, midline, and period.

One sample for each option (e.g., further develop, apply, introduce) is included below. The task plans use the task planning guide provided in Smith, M., Steele, M. D., & Sherin, M. G. (2020). *The 5 Practices in Practice: Successfully Orchestrating Mathematical Discussions in Your High School Classroom*. NCTM.

A sample task that was designed **to further develop** is on pages 4-10.

A sample task that was designed **to apply** is on pages 11-19.

A sample task that was designed **to introduce** is on pages 20-24.




An example task that was designed **to further develop** understanding of amplitude, midline, and period related to sine functions and their graphs:

### Lesson Topic: Sine Graphs: Amplitude, Midline, Period

*Task Planning Guide for Sample 1*

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| <p><b>Learning Goals:</b><br/> <i>What understandings will students take away from this lesson?</i></p> <p>Students will be able to determine the amplitude of a given function or a graph.<br/>         Students will be able to determine the midline of a given function or a graph.<br/>         Students will be able to determine the period of a given function or a graph.<br/>         Students will be able to create a sine function that accurately describes a given graph.</p>   | <p><b>Evidence:</b><br/> <i>What will students say, do, produce, etc., that will provide evidence of their understandings?</i></p> <p>We will be able to understand if students can determine amplitude, midline, and period of a given function if the students are able to point out which numbers are the amplitude, midline and period, and the students are also able to describe why their answer is correct.</p> <p>We will be able to understand if students can determine the amplitude, midline, and period of a given graph if they are able to accurately point out each number and describe why their answer is correct.</p> <p>We will be able to understand if students can create a function that accurately describes a given graph if their function works and they can explain why it does.</p> |
| <p><b>Task</b><br/> <i>What is the main activity that students will be working on? Briefly describe it and tell us where to find it (e.g., attached, link).</i></p> <p>Students will be mainly working on pages 8-17 and using the information they have learned about amplitude, period, and midline to accurately describe a graph or function. They will then write an accurate function combining the information they have learned about amplitude, midline, and period. I want students to further develop their understanding. Pages 1-7 is set to be a very quick review of amplitude, midline, and period and can be used as an opportunity to clear up any misconceptions very quickly. Pages 18-20 are set to be challenges for students to</p> | <p><b>Instructional support – tools, resources, materials</b><br/> <i>What tools or resources will be made available to give students entry to, and help them reason through, the activity?</i></p> <p>Each student (or pair of students) needs a computer with internet access.</p> <p>In addition, we will use the Desmos activity and your brain ;-)</p>  |



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| <p>complete if there is time. The challenges are meant for students to actually sketch a graph of a given function.</p> <p> <a href="#">More Sine Graphs Activity Link</a></p>  |   |
| <p><b>Prior Knowledge</b><br/><i>What prior knowledge and experience will students draw on in their work on this task?</i></p> <p>Prior knowledge of amplitude, period, and midline (there will be a mini review at the beginning of the activity, but students need to understand more).</p> <p><b>Essential Questions</b><br/><i>What are the essential questions that I want students to be able to answer over the course of the activity?</i></p> <p>What are amplitude, midline, and period?<br/>How can I use what I know about amplitude, midline, and period to accurately describe a graph?</p>  | <p><b>Task Launch</b><br/><i>How will you introduce and set up the task to ensure that students understand the task and can begin productive work, without diminishing the cognitive demand of the task?</i></p> <p>I would tell students that we are working on furthering their understanding of amplitude, midline, and period. The first 9 pages will give a small and quick review of those three terms before we start working with functions and graphs of functions. The first 9 pages should take about 5 minutes max to work on. After the 9 pages are done, I would make sure that there were no misconceptions by looking at the Teacher Dashboard. If there was no misconception, students will go into their small groups to start working on pages 10 til the end. If there was a misconception, I would address it before sending students to their groups.</p> |
| <p style="text-align: center;"><b>Anticipated Solutions and Instructional Supports</b></p> <p><i>What are the various ways that students might complete the activity? Be sure to include incorrect, correct, and incomplete solutions.</i></p> <p><i>What questions might you ask students that will support their exploration of the activity and bridge between what they did and what you want them to learn? These questions should assess what a student currently knows and advance him/her toward the goals of the lesson. Be sure to consider questions that you will ask to students who can't get started as well as students who finish quickly.</i></p> <p style="text-align: center;"><i>Use the monitoring chart (below) to provide the details related to Anticipated Solutions and Instructional Support</i></p> |   |
| <p><b>Sharing and Discussing the Task</b></p>  |   |
| <p><b>Selecting and Sequencing</b><br/><i>Which solutions do you want students to share during the lesson?</i></p> <ol style="list-style-type: none"> <li>1. Students who accurately determine the amplitude, period, and midline.</li> <li>2. Students who have the right numbers, but did not determine correctly which</li> </ol>   | <p><b>Connecting Responses</b><br/><i>What specific questions will you ask so that students</i></p> <ul style="list-style-type: none"> <li>• <i>Make sense of the mathematical ideas that you want them to learn?</i></li> <li>• <i>Make connections among the different strategies / solutions that are presented?</i></li> </ul>  |



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| <p>number goes with amplitude, period, or midline.</p> <ol style="list-style-type: none"> <li>Students who create an accurate function to describe a given graph.</li> <li>Students who create a function that is on-the-right-track of becoming correct.</li> </ol> <p><i>In what order? Why?</i></p> <p>I would talk about numbers 1 &amp; 2 first. Students must be able to determine which parameter determines what the amplitude, midline, and period would be. This is because I need to make sure that my students understand which numbers correspond to amplitude, midline, and period correctly. It would be a great time to clear up any misconceptions. Then, I would talk about numbers 3 &amp; 4. I want students to be rewarded for knowing what numbers need to be involved in writing an accurate function, but I also want students to know the correct answer and how to get there.</p> | <p>What do you find that is similar between these determinations?</p> <p>What do you find that is similar between these functions?</p> <p>What is different (between these determinations/between these functions)?</p> <p>How do we know which determination is correct?</p> <p>How do we know which function is correct?</p> |
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## Monitoring Chart

*Monitoring Chart Color Coding by Desmos Activity Page*

|                          |
|--------------------------|
| Activity Pages 1-8       |
| Activity Pages 10 and 11 |
| Activity Pages 12-14     |

*Monitoring Chart for Sample 1*

| Anticipated Solutions / Strategies  | Instructional Support  |   | Who? | Select / Sequence |
|---|--|---|------|-------------------|
|   | Assessing Questions  | Advancing Questions   |      |                   |
| Students can accurately determine amplitude, midline, and period given a function   | Even though the activity asked students to explain their answer, I would ask: How do you know that your answers accurately describe the functions' parameters? | N/A   |      |                   |
| ***<br>Students can accurately determine one or two parameters (amplitude, midline or period), but not the other one or two parameters given a function | How did you know that (parameter) was (student answer)?<br>What other parameters do we need?   | Go back to the sliders on page 1 to see if you can determine what other parameters we need and what numbers correspond to those parameters. |      |                   |
| Students do not know how to start determining the amplitude, midline, or period given a function  | What information do we have?<br>What information do we know?   | Go explore the sliders again on page 1.   |      |                   |
| Students can accurately determine amplitude midline and period given a graph  | Even though the activity asks student to explain their answer, I would ask: How do you know that your answers accurately describe the graph's parameters?      | N/A   |      |                   |



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| Students can accurately determine one or two parameters (amplitude, midline, period), but not the others | How did you know that (parameter) was (student's answer)?<br>What other parameters do we need?<br>What numbers do you think need to be involved in your answer based on the graph? Why? | After students know the numbers that need to be involved but are confused on which parameter corresponds with that number, "Where did you find that number and how does it relate back to amplitude, midline, or period?"<br>If students are still struggling, encourage them to go back to explore the sliders on page 1 again. |  |  |
| Students don't know how to start determining the amplitude, midline, or period given a                   | What do you know based on the graph?<br>What do we need to know?<br>How can you find (parameter) based on the graph?  | How can we use what we learned about amplitude, midline, and period with the sliders and the descriptions on pages 1-7 to determine what the amplitude, midline, and period is for the graph? Go explore the sliders on page 1 if needed.  |  |  |
| Students can create a function that accurately describes a graph   | Even though the activity asked students to explain their answer, I would ask: How do you know that your function accurately describes the graph?  | N/A  |  |  |
| Students can determine one or two parameters that they need, but not the other one or two                | How did you know that (parameter) was (student answer)?   | If you know the numbers that need to be involved in your function, what numbers  |  |  |



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|   | <p>What other parameters do we need?</p> <p>What numbers do you think need to be involved in your function based on the graph? Why?</p>  | <p>correspond with amplitude, midline, and period?</p> <p>Where does the amplitude go in the function?</p> <p><math>(y = a \sin(bx) + k)</math></p> <p>Where does the midline go in the function?</p> <p>Where does the period go in the function?</p>   |  |  |
| Students do not know how to start creating a function | <p>What do you know so far?</p> <p>What do we need to create an accurate sine function?</p> <p>What numbers do you think need to be involved in your function based on the graph? Why?</p> | <p>Encourage them to look back on their previous work on the activity to see if that helps any.</p>  |  |  |
| Students says that amplitude is a negative number.    | <p>What made you say the amplitude was negative?</p>   | <p>What does the amplitude describe?</p> <p>If the students says “height”, ask them about their negative number again in the context of height.</p> <p>If the student does not know, either tell them the definition or make them go back to the slides that briefly talk about amplitude.</p> |  |  |



### Summarizing Why

The reason why I designed this task the way that I did is to give students plenty of opportunity to practice amplitude, midline, and period with the sine function. I wanted students to be able to tell me what the amplitude, midline, and period is for a given function and for a given graph. The reason why my task is a good mathematical task is because it: creates opportunities for students to be right and wrong in different, interesting ways; gives feedback that attaches meaning to student thinking; and uses a variety of resources. I create an opportunity for students to be right and wrong in different and interesting ways by asking them to create functions that would accurately represent a given graph. After students create their functions, I will allow them to check their work using the graphing calculator provided on the next slide (page 15). This also gives students feedback to their thinking. Allowing students to check their functions after they created them to see if they accurately describe a graph will allow for students to wonder where they went wrong in creating their function. Instead of telling students that they are wrong, they are given the opportunity to compare the graph they created with the function to the graph that was assigned to them. I also gave students feedback on the first 7 slides by programming the activity to tell the students a little bit about the answer they chose & pose a thinking question if their answer was incorrect. I then use a variety of resources by allowing students to type/explain their answer, explore with sliders, use graphing calculators, and the sketching tool at the end for the challenges. Asking students to type and explain their answer allows me to get more assessment data on how students are doing with this topic. This will also allow me to take screenshots of their answers and share with the class if I want to have a discussion. Exploring with sliders allows students to get a better understanding of what each parameter does to the sine function. This also allows students to have a review of what they have learned prior. Using graphing calculators will give students the opportunity to visibly see whether or not their function that they created is an accurate description of the assigned graph. Lastly, the sketching tool at the end of the activity for the challenges gives students an opportunity to tie together what they have learned with functions and graphs and attempt to sketch a graph that would describe the given function. This activity is meant to push students to become better with amplitude, midline, and period. If students can accurately determine and describe the 3 parameters given a function and a graph, and then accurately make their own functions, they will be able to do other activities that make them think more deeply about what they know about the 3 parameters (the Burning Daylight Activity on Desmos for example). I wanted students to have a better understanding of the 3 parameters before jumping into more applying activities.



An example of a task that was designed for students **to apply** their knowledge of amplitude, midline, and period to a real context through modeling.


**Lesson Topic:** Sine Graphs in Context- Applying Amplitude, Midline, and Period

*Note: Written explanation of design choices/alignment with principles is after monitoring chart*

*Task Planning Guide for Sample 2*

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| <p><b>Learning Goals:</b><br/><i>What understandings will students take away from this lesson?</i></p> <ul style="list-style-type: none"> <li>• Students will be able to recognize situations that can be modeled by sine functions.</li> <li>• Students will be able to compare/contrast and interpret sine functions in context.</li> <li>• Students will be able to describe the parameters of sine functions with both informal and formal vocabulary.</li> <li>• Students will be able to use the graphs of sine functions to make predictions in context.</li> <li>• Given the graph of a sine function, students will be able to determine and interpret the amplitude, period, and midline.</li> </ul> | <p><b>Evidence:</b><br/><i>What will students say, do, produce, etc., that will provide evidence of their understandings?</i></p> <ul style="list-style-type: none"> <li>• Students will be able to explain (in writing or aloud) what amplitude, period, and midline mean in the context of monthly temperatures, in their responses and explanations in the activity.</li> <li>• Students will be able to identify and describe the differences between sine functions/graphs and explain them in the context.</li> <li>• Students will be able to explain why they made their predictions, based on the graphs of the previous slides in the context of average temperatures.</li> </ul> |
| <p><b>Task</b><br/><i>What is the main activity that students will be working on? Briefly describe it and tell us where to find it (e.g., attached, link).</i></p> <p>In this activity, students use sine graphs to model temperature data for two US cities (New York, NY and Orlando, FL). They explore the parameters of a sine graph (amplitude, midline, and period) and interpret them in the context of monthly temperatures in the USA.</p> <p>Students will use their knowledge of a midline to estimate the average yearly temperature in both cities. Students will also use what they learned from the NY and FL models to predict</p>   | <p><b>Instructional support – tools, resources, materials</b><br/><i>What tools or resources will be made available to give students entry to, and help them reason through, the activity?</i></p> <p>Each student (or pair of students) needs a computer with internet access.</p> <p>In addition, we will use a Desmos activity that allows students to sketch and interact with pre-existing graphs to support student application of the parameters of sine functions in context.</p>   |



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| <p>what a model for the monthly temperatures in Charlotte, NC looks like.</p> <p> <a href="#">Heat on the East Coast Activity Link</a></p>   |   |
| <p><b>Prior Knowledge</b><br/><i>What prior knowledge and experience will students draw on in their work on this task?</i></p> <p>Students will build on their knowledge of parameters of sine graphs (specifically amplitude, period, and midline) and how these parameters affect the graphs of sine functions, through interpretation and application.</p> <p><b>Essential Questions</b><br/><i>What are the essential questions that I want students to be able to answer over the course of the activity?</i></p> <p>How can I use a sine function to model a situation in a real life context?</p> <p>How can I describe the parameters of a sine function in a real life context?</p>  | <p><b>Task Launch</b><br/><i>How will you introduce and set up the task to ensure that students understand the task and can begin productive work, without diminishing the cognitive demand of the task?</i></p> <p>Depending on the time of year, I plan on launching the task by remarking about the current weather outside (either “Wow, it sure is hot today” or “Brrr, it’s cold outside!” or something of that sort). I won’t elaborate on that point immediately, but then talk about the parameters of sine graphs that they worked on before. I will quickly review amplitude, period, and midline by asking students to volunteer and explain them to me. Once I review the parameters so they are on the same page and understand we are working with sine graphs, I will give them the task, and let them begin.</p> |
| <p><b>Anticipated Solutions and Instructional Supports</b><br/><i>What are the various ways that students might complete the activity? Be sure to include incorrect, correct, and incomplete solutions.</i></p> <p><i>What questions might you ask students that will support their exploration of the activity and bridge between what they did and what you want them to learn? These questions should assess what a student currently knows and advance him/her toward the goals of the lesson. Be sure to consider questions that you will ask to students who can’t get started as well as students who finish quickly.</i></p> <p><i>Use the monitoring chart (below) to provide the details related to Anticipated Solutions and Instructional Support</i></p> |   |
| <p><b>Sharing and Discussing the Task</b></p>   |   |
| <p><b>Selecting and Sequencing</b><br/><i>Which solutions do you want students to share during the lesson?<br/>In what order? Why?</i></p> <p>I will anonymize student responses to begin, as to not call anyone out or make anyone</p>   | <p><b>Connecting Responses</b><br/><i>What specific questions will you ask so that students</i></p> <ul style="list-style-type: none"> <li><i>Make sense of the mathematical ideas that you want them to learn?</i></li> <li><i>Make connections among the different strategies / solutions that are presented?</i></li> </ul>  |



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| <p>look bad when student responses are shared with each other.</p> <p>I will pace them to slides 1 and 2, and after there are many responses for Slide 2, I will showcase student responses. I want to highlight answers for both Orange and Purple, and highlight reasonings behind each to spark discussion among the students.</p> <p>After having a full group discussion with Slide 2, I will extend the pacing through slide 5, to let the students create their graphs for Orlando and New York, then compare and contrast them. While they are working on this, I will make it clear that no equation will perfectly match the data, so I just want them to find good models that fit it the best they can.</p> <p>Once they finish through slide 5, I will showcase multiple student models for Orlando and let students discuss similarities and differences. Then, I will showcase multiple student models for NYC and do the same. For slide 5, I will highlight informal and formal responses, and try to connect informal responses to key vocabulary like period, amplitude, and midline.</p> <p>I will extend pacing through slide 7, then discuss with students again. I will highlight variation in the average temperatures and allow students to explain their answers, and I would like to conclude the discussion of this side by allowing them to see that the midline should demonstrate the average of the data set.</p> <p>I will let students work through the remaining slides, and then highlight student responses and let them explain the similarities and differences they found. I want to focus on connecting the equation to the graph of the function, and especially interpret the parameters in this context.</p> <p>The period should be the same, because every data set is working off a span of 2</p> | <p>I want to make sure to make connections between the informal and formal language of:</p> <ul style="list-style-type: none"> <li>• Descriptions of the equations/models and the period, amplitude, and midline</li> <li>• Interpreting the parameters in the terms of average temperatures.</li> </ul> <p>I want to especially make connections on the final discussion at the end between:</p> <ul style="list-style-type: none"> <li>• The equation and the graph/model</li> <li>• The meaning of the period and what it means in this context, and why it is the same.</li> <li>• The meaning of amplitude, what it means in this context, and why it differs.</li> <li>• The meaning of period, what it means in this context, why it differs, and how it can be used to find averages.</li> </ul> |
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years, with a period being every year. The amplitudes demonstrate the range of the temperatures, from the hottest months to the coldest months, and highlight that Orlando has a much smaller amplitude and ask why that might be. Finally, I want to reinforce that the midline is the average or middle of the data.



## Monitoring Chart

Monitoring Chart for Sample 2

| Anticipated Solutions / Strategies   | Instructional Support   |   | Who? | Select / Sequence |
|--|---|---|------|-------------------|
|  | Assessing Questions   | Advancing Questions   |      |                   |
| Slide 3/4:<br>Made a sine function whose graph lines up well with the data   | Have you had any trouble making an equation?<br><br>How did you decide upon this equation?      | What do the values you chose for your parameters mean?<br><br>What about the data set made you decide that a sine function would fit it best?   |      |                   |
| Slide 3/4:<br>Made a sine function whose graph is too low/high for the data  | Have you had any trouble making an equation?<br><br>What have you tried so far?                 | Have you tested the different parameters of your equation? Which parameter moves your graph up and down?  |      |                   |
| Slide 3/4:<br>Made a sine function whose graph is too far left or right for the data<br><br>**(Phase Shift, they do not know about this yet)** | Have you had any trouble making an equation?<br><br>What have you tried so far?                 | Have you tested the different parameters of your equation?<br><br>This one is one we have not explored much yet. In other function families, how did you move the graph of a function left or right? Have you tried using that method here? |      |                   |
| Slide 3/4:<br>Made any function other than a sine function.  | Have you had any trouble making an equation?<br><br>What have you tried so far?                 | What do you notice about the data?<br><br>What type of function might fit the data? (Give examples for them to think about: linear, quadratic, exponential, logistic, sine, and let them explain why they would/wouldn't work)              |      |                   |
| Slide 6:<br>Average temperatures are within reasonable range, by using midline   | How did you get this answer?<br><br>Does this answer make sense in the context? Why or why not? | Could you use the midline to determine the average of any data set modeled by a sine function? Why or why not?  |      |                   |



|  |  |  |  |  |
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| Slide 6:<br>Average temperatures are within reasonable range, by other methods (that students listed on slide 7)       | How did you get this answer?<br><br>Does this answer make sense in the context? Why or why not?  | Would this method of finding the average work for any data set modeled by a sine function? Why or why not?   |  |  |
| Slide 6:<br>Average temperature is unreasonable.   | How did you get this answer?<br><br>Does this answer make sense in the context? Why or why not?  | What does the average of a data set mean, especially in the context of temperatures?<br><br>What is one way we might find this average based on the parameters of a sine function?   |  |  |
| Slide 10:<br><br>Mainly discussion, less strategy based:<br><br>Similarities/Differences in period, amplitude, midline | What similarities do you notice between the graphs and equations of each city's temperatures?<br><br>Are some parameters the same? How do you know? What might this mean in the context? | What does it mean, in the context of average temperatures, for each city to have the same period?<br><br>Why, in the context, are the amplitudes different? Which city has the largest amplitude? Which city has the smallest amplitude? What does this mean?<br><br>What does the midline mean in the context of temperatures? Why is one city's midline higher/lower than another? What can the midline tell you about a city's temperature? |  |  |



## Written Explanation of Design Choices and Alignment with Design Principles for a Good Technology Task

- Starting on Slide 1, I decided to make the cities we are working with initially Orlando and New York, because they are both on the east coast where we are, and they are major cities that high school students have hopefully heard of even if they have not been to them. I included a small picture showing the locations on a map, and I made sure to include Charlotte on both maps for reference later on. I also believed having some sort of visual representation of the cities we are looking at would help students understand the context.
- With Slide 2, I decided to make a simple comparison between the two data sets by letting students decide which one is which. I specifically made them contrasting colors, with Orlando being Orange because Florida tends to be hotter and orange is a warm color, while New York is purple since NY tends to be colder and purple is a cool color. This is not obvious to students by any means, but once the graphs are revealed, students will hopefully subconsciously distinguish between the two more easily based on the color difference.
- With Slides 3 and 4, I wanted to separate the two data sets so that it was not convoluted or confusing for students to look at when they create equations for the functions. I also wanted to use data sets that would not be *perfect* fits for a sine function, but that would be best represented by a sine function, to show that even if data does not line up perfectly it can still be modeled by a specific function (in this case, sine).
- With Slide 5, I compare the two equations that students used to model the data, and at this point I wanted to let students start noticing and explaining the parameters of the graphs, in either informal or formal language. I can use the informal responses to make connections TO the formal responses, so that students can associate their basic knowledge with the specific parameters and their definitions.
- Slides 6 and 7 work with the same graphs, but indirectly allow students to focus on midline and its meaning in the context. Instead of blatantly telling students, “Hey, the midline is the center and can be interpreted as an average of the data set,” I would rather let them come to that conclusion on their own to enhance the cognitive demand of the task. Then, once I see some student responses, I can use our discussion to come to that conclusion.
- Slides 8 and 9, I wanted to make the activity more relatable and personal to the students, so that they could engage more with it. Since this is a class taught in Charlotte, NC, I presumed that most of these students live in Charlotte, NC, or the surrounding area. Students will be able to relate it to their personal experiences with the weather here, and see how the graphs and functions they’ve been working with manifest themselves in their everyday lives. This part is also more interactive, as it allows students to sketch their own graph and then have an exciting reveal to see how accurate their prediction was to the data.



(This is my personal favorite part of the task, and I think the students will find it exciting as well).

- Finally on Slide 10, we wrap up the task by seeing the sine functions that best fit the data, and we get to interpret those in the context of the situation. This is the major discussion point of the task, as it allows the teacher to connect the parameters in context to both the equations and the graphs, tying all of it together. Even though the task itself does not blatantly ask, “What about the parameters?”, it once again leaves it open for the teacher to use student responses and informal/formal language to connect the parameters to the context, making the full class discussion that much more effective.

### How it Relates to the Principles of a Good Task/Good Design

- Looking first at PTMT’s design principles in “Planning and Teaching Algebra with Technology Chapter 2:”
  - **Ease of Use:** This task is very straightforward with what it is asking, and I specifically focused on the color differences between Orlando, New York, and later Charlotte, so that the various functions/data sets on the same graph do not get confusing at all. I have personally tested my task with correct and incorrect responses to make sure that students do not encounter roadblocks in their progress throughout it.
  - **Presentation:** The task is clear and not wordy, as I did not include big paragraphs or chunks of text anywhere. Most of the text is limited to one sentence or question, and the important descriptive terms for the context are repeated often for clarity. Links between the representations are obvious, as I specifically have slides that allow students to distinguish which data set belongs to which city, and the color coordination is maintained throughout. Whenever the graph is needed for interaction, such as drawing sketches or lines, the graph is set up to enable students to do so. On slides where students are only supposed to observe the graphs and take note of what they notice, the tools for drawing/interacting with the graphs are hidden.
  - **Support for Problem Solving:** As this task is largely based on making observations, interpreting parameters, and making predictions, there are not many occasions when students have to specifically “problem solve.” When students need to create equations to model the data set, the graph changes based on their input, so they can see whether their equation is getting closer to the data set as they type, and they can see what specific part of their equation affects their graph in a certain way. Later in the task, when students are asked to make a predictive sketch for Charlotte’s temperatures, they can use the information they have gathered as well as their personal experiences to influence their predictions, and then compare their predictions with the real data set.
  - **Low/High Cognitive Demand:** I believe that this task is entirely made of high-cognitive demand tasks, as every task is either “Procedures with connections” or “Doing mathematics.” Students have to write equations



and make connections between the parameters and the contexts, and students are asked to explain their reasoning for every answer they give. Each slide actively engages the students' thinking and sparks a discussion for the class.

- Also looking at “The Desmos Guide to Building Great (Digital) Math Activities v2.0”:
  - **Opportunities to be right and wrong in different, interesting ways:** The only instance in this activity where it tells the students which answer is correct is when students are asked to decide which graph models which city's temperatures, because that is necessary for understanding the graphs going forward. Every other time the student creates their own answer, they are not told whether they are right or wrong. Instead, students see how their equations are modeled as they are creating them, allowing for trial and error until they personally decide they've made the best model they can make. Almost every answer after that point is based on each model that the students made for themselves, and gives the opportunity for students to explain and defend their answer in the context.
  - **Give feedback that attaches meaning to student thinking:** As mentioned before, the task does not tell them how accurate their equations are to the data set, but instead models for them what their equation would look like and allows them to change and adjust it as needed. Additionally, the entire activity is placed in context, so that students do not see numbers as just numbers, but instead, “The average temperature in Orlando is 76 (for example), so Florida is pretty hot year round!” or “The amplitude of New York's model is around 21, so there is a pretty big difference between the temperature in the summer and the temperature in the winter.” The task takes the material (sine graphs) and puts it into a context that students can understand and experience, especially when the task starts talking about Charlotte.
  - **Interrupt our biases:** When creating this assignment, I wanted to make sure it was something that students, ALL students, could relate to. Not every student gets to go on vacation or travel, so I did not want to pick locations that were super exotic that students could not even reasonably imagine. I decided to stay pretty close to home with New York and Orlando, because even if they are a few hours away, they are still on the east coast and high school students likely will have heard of them at least regardless of their background or identity. Then, as I mentioned before, I wanted to take what the students have done and bring it back home to Charlotte, so that students could make a direct connection from the math to their lives.




An example of a task that was designed to introduce phase shift to go along with amplitude, midline, and period.

## Lesson Topic: Introduction to Phase Shift

*Task Planning Guide for Sample 3*

|  |   |
|--|---|
| <p><b>Learning Goals:</b><br/><i>What understandings will students take away from this lesson?</i></p> <ul style="list-style-type: none"> <li>Students will be able to determine the phase shift of a sine function given the graph.</li> <li>Students will be able to determine the phase shift of a sine function based on the equation of the function.</li> <li>Students will be able to determine the function equation, given the amplitude, midline, and period, and phase shift.</li> <li>Students will be able to determine the function equation given the graph of the function.</li> </ul>   | <p><b>Evidence:</b><br/><i>What will students say, do, produce, etc., that will provide evidence of their understandings?</i></p> <ul style="list-style-type: none"> <li>Students will be able to input responses, both numerical and explanations, into the desmos activity.</li> <li>Students will be able to engage in a whole class discussion and elaborate on their experiences and thoughts while completing the desmos activity.</li> <li>Students will be able to input their answers into the desmos activity that correspond to the given question.</li> <li>Students will be able to explain through the desmos activity or verbally the significance of phase shift and how to find it using a graph and the function equation.</li> </ul> |
| <p><b>Task</b><br/><i>What is the main activity that students will be working on? Briefly describe it and tell us where to find it (e.g., attached, link).</i></p> <p>Students will work through a desmos activity designed to review what they have previously learned about amplitude, midline, and period, to build off of that knowledge and introduce them to phase shift. Students will explore phase shift first through sliders. They already have explored amplitude, midline, and period this way, so the only new slider, <math>h</math>, is responsible for phase shift. Students will then work through finding the phase shift of functions when they are graphed with their parent function and when they are only given an equation. Then students will create an equation with a given phase shift and explore a function with a phase shift of <math>2\pi</math>. Students</p> | <p><b>Instructional support – tools, resources, materials</b><br/><i>What tools or resources will be made available to give students entry to, and help them reason through, the activity?</i></p> <p>Each student (or pair of students) needs a computer with internet access.</p> <p>In addition, we will use pencil and paper for students to work through calculations or sketch graphs as they solve problems to get conceptual understanding.</p>   |



|   |   |
|---|---|
| <p>are asked to explain their reasoning throughout this task. The task finishes with a card sort where students have to match the graph to its phase shift and a challenge exercise where students are no longer given the parent function.</p> <p> <a href="#">Phase Shift Activity Link</a></p>  |   |
| <p><b>Prior Knowledge</b><br/><i>What prior knowledge and experience will students draw on in their work on this task?</i></p> <p>Students will build upon their prior knowledge of amplitude, midline, and period as well as their prior knowledge about function transformations.</p> <p><b>Essential Questions</b><br/><i>What are the essential questions that I want students to be able to answer over the course of the activity?</i></p> <p>How can phase shift be found from a graph of a function?</p> <p>How can phase shift be found from a functions equation?</p>   | <p><b>Task Launch</b><br/><i>How will you introduce and set up the task to ensure that students understand the task and can begin productive work, without diminishing the cognitive demand of the task?</i></p> <p>This task starts with a warm up where students will be asked to describe a function given what they recall about amplitude, midline, and period. They will then transition to the next few slides which briefly explains these concepts again so that all students will be able to start this task and draw on this knowledge regardless of their previous experiences. This will be a whole class discussion also, so students can learn from what their classmates remember if they themselves cannot recall a lot.</p> |
| <p><b>Anticipated Solutions and Instructional Supports</b><br/><i>What are the various ways that students might complete the activity? Be sure to include incorrect, correct, and incomplete solutions.</i></p> <p><i>What questions might you ask students that will support their exploration of the activity and bridge between what they did and what you want them to learn? These questions should assess what a student currently knows and advance him/her toward the goals of the lesson. Be sure to consider questions that you will ask to students who can't get started as well as students who finish quickly.</i></p> <p><i>Use the monitoring chart (below) to provide the details related to Anticipated Solutions and Instructional Support</i></p> |   |
| <p><b>Sharing and Discussing the Task</b></p>   |   |
| <p><b>Selecting and Sequencing</b><br/><i>Which solutions do you want students to share during the lesson?</i></p> <p>I want students to work through slides 1-4 as a warm up that the whole class goes over.</p>   | <p><b>Connecting Responses</b><br/><i>What specific questions will you ask so that students</i></p> <ul style="list-style-type: none"> <li><i>Make sense of the mathematical ideas that you want them to learn?</i></li> </ul>  |



|  |   |
|--|---|
| <p>Then work in groups on 5-13. From these, I want the center a whole class discussion on 7, 9, and 10.</p> <p><i>In what order? Why?</i></p> <p>I will order the discussion in the order of the slides pulling from student work that is more informal and ending with more formal. I want to acknowledge all students work and structure the discussion in a way that is logical to follow and move from one idea to the next.</p> | <ul style="list-style-type: none"> <li>• <i>Make connections among the different strategies / solutions that are presented?</i></li> <li>• What did you notice about slider h?</li> <li>• What was your reaction to slider h?</li> <li>• While working through the activity, did you make any connections to past topics we've covered?</li> <li>• What might be the benefits of using a graph to find phase shift over just the equation?</li> <li>• What might be the benefits of using the equation to find phase shift over just the graph?</li> <li>• What is significant about a function with a phase shift of <math>2\pi</math>?</li> </ul> |
|--|---|



### Monitoring Chart

*Monitoring Chart for Sample 3*

| Anticipated Solutions / Strategies   | Instructional Support   |   | Who? | Select / Sequence |
|--|---|---|------|-------------------|
|  | Assessing Questions   | Advancing Questions   |      |                   |
| Students will confuse vertical and horizontal shift.                                       | What effect does $h$ have on the function?  | How does $h$ affect the graph compared to $k$ ?   |      |                   |
| Students will focus on period and not phase shift.   | What effect does $h$ have on the function?<br><br>What is your strategy for finding phase shift on a graph?               | What is the difference between what we learned previously about the period of a function and the phase shift of a function?   |      |                   |
| Students will not recognize that the function has a negative in front of $h$ .             | What effect does $h$ have on the function?<br><br>Which direction does $h$ move the function based on the equation alone? | Does this match up to what we can see in the graph when we move the $h$ slider? Why or why not?   |      |                   |
| Students will sketch the entire sin function on slide 13 to help them answer the question. | What was your strategy to find the phase shift of this function?  | Is there a way to do this without sketching the entire sin function?<br><br>Can this be done in less steps?<br><br>What is one key point that we could look at to help us figure out the functions phase shift. |      |                   |



### Written explanation of design choices:

In this task I chose to start with a warm up that allows all students the ability to engage with the task ahead. No matter students' previous experiences, they can start the task. I also ask students for an informal analysis of slider  $h$  before explaining phase shift, which is one of the points made in “The Desmos Guide to Building Great (Digital) Math Activities”. This gets students ready for more formal analysis later on. Another key point explained in this article is to “keep expository screens short, focused, and connected to existing student thinking”. When explaining phase shift, the slide was kept brief and to the point while also asking students to find the phase shift given a graph and equation, both of which they had just been exploring. This also connects to one of the design principles which is ease of use. I tried to bold key terms to make them stand out and create clear concise instructions. Students should not be held back from engaging with the material because they do not know how to use the technology. This is also heavily connected with another design principle, presentation. Additionally, support for problem solving is present in the warm up and through the dynamic graphs. Students are also never told how to solve a problem, they can go about this in their own ways, using their own strategies.