[Module 7 Overview Document]

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	Homework	7.1 Engage in Introduction to the Sine Graph Desmos Task
	Day 1	20 min	7.1 Discussion Optional: Extend the Discussion: Task Design
		35 min	7.2 Launching a Technology-Mediated Math Task
		20 min	7.3 Noticing Student-Teacher Interactions
		Homework	7.4 Monitoring Student Thinking: Introduction to the Sine Function
	Day 2	15 min	7.4 Discussion
		20 min	7.5 Noticing Student Thinking about Amplitude
		40 min	7.6 Noticing Student Thinking about Period
	Day 3	40 min	7.7 Designing a Sequence of Tasks (optional project)

7.6 Facilitation Notes

The students in this video are not from the class in which the videos included in tasks 7.2– 7.4 were recorded. However, it is the same lesson and was implemented very similarly in a face-to-face setting in Fall 2021 (Note: You will see students wearing face masks in the classroom). The students had previously learned about many function families (e.g., linear, quadratic, exponential, absolute value) through similar parameter explorations but had not been formally introduced to the sine function prior to this task. However, students have seen that the sine function creates a "wave" through looking at models of a cart moving around a Ferris Wheel over time (like in Module 3: Desmos Function Carnival).

If teachers have completed 7.4 Noticing Student-Teacher Interactions in a Technology-Mediated Environment, they have seen short snippets of students thinking about various parts of this task. Here they will have the opportunity to examine authentic student work on the task. They will see a pair of students who are working collaboratively on one laptop. They will see the students' computer screen (i.e., their technology engagement) and listen to them discuss their ideas. Specifically, this video clip is focused on a pair of students, Rhian and Shakira, making sense of the relationship between slider b and the period of the sine function $y = a\sin(bx) + k$. Specifically, teachers will see Rhian and Shakira at two different times in the course of working on the 7.1 Introduction to the Sine Graph task, both clips focused on their sensemaking of period. In the first clip the students are determining the period from a graph, so have developed a strategy using the difference between two consecutive maximum or minimum points (as the definition suggests), but later find this strategy is not helpful if they need to determine the value of b.

We recommend that teachers work in pairs on this task. Providing teachers with the transcript of the video to refer to as they are responding to the task prompts is helpful.

After the teachers have worked in pairs on 7.6, it will be necessary to facilitate discussions. One suggestion is to focus this discussion on a subset of the questions such as Q7 and Q8. If possible, have the teachers put their responses to Q7 and Q8 onto a group sharing platform (e.g., Jamboard) and have them review each other's responses before facilitating a whole group discussion modeling the select and sequence component of the 5 practices using the teacher's written responses.

7.6 Sample Responses
Noticing Student Thinking about Period

Context

Ms. Fye is using the Introduction to the Sine Graph Desmos Task in an in-person class.

This is an introduction task focused on reasoning about the relationship between the equation of a sine function and characteristics of the sine graph. Ms. Fye designed the task knowing that her students had explored the relationship between function structure and their graphs by varying the parameters for many different function families. With that in mind she had the following learning goals:

- Students will recognize the connection between the structure of a sine function equation (i.e., $y = a\sin(bx) + k$) and its related graph with respect to amplitude, midline, and period. Specifically,
    - Amplitude is $|a|$
    - Midline is $y = k$
    - Period is $\frac{360}{|b|}$

Specific performance goals include:

- Given a sine function equation, students will determine the amplitude, period, and midline without graphing.
- Given the amplitude, midline, and period, students will determine the function equation.
- Given the graph of a sine function, students will determine the amplitude, period, and midline.
- Given the graph of a sine function, students will determine the function equation.

In the video clip that follows, Rhian and Shakira are working on page 10 of the Desmos task. On this page, they are given the amplitude, midline, and period and asked to determine the equation of sine function.

Watch the video clip and listen carefully to what the students are saying and the ways they are interacting with the technology. Then, rewatch as needed to answer the questions below.

Rhian and Shakira Exploring Period of the Sine Function (Clip 1)

Q1. What is something smart that Rhian and Shakira said or did in this clip?

- I like that they started by putting the base equation there so they could add the stuff to it. That was a smart way to start.
- They used the graph on page 5 and changed the sliders to match what they were given to help them think about their response.
- Used the graph on page 5 to try to make a function with a period of 60.

Q2. Attend to Rhian and Shakira's strategy(ies) for determining the value of b when they are given the period.

- They used the graph on page 5 and changed the value of b until the difference between two consecutive maximum points was 60.
- They used trial and error with the sliders to find a value for b that resulted in a graph with a period of 60.
- They used the definition of period and the graph on page 5 to find a value of b that would result in a period of 60.

Q3. What do you know about their current understanding of period? Provide evidence from the clip to justify your answer.

- Rhian and Shakira understand the definition of period as the distance from one "peak" to the next "peak". This is evident in the way they use the graph on page 5 to find a function that has a period of 60. They change the value of b and check that distance until they find one with a distance of 60.
- They understand period as distance and they know that b affects it (we know this because they use trial and error to change b until they find an example with the right period). I don't know if they know period as the horizontal length of a complete cycle (the first part of the definition).
- I think they understand the 2nd part of the definition that is provided on page 8, but not the first part. I think that because rather than thinking about the length of the cycle to find b they went back to the graph and found the distance between the peaks and changed the value of b until they got what they needed.

Q4. What questions would you ask to further assess their understanding and support them in making connections?

- After they found a b that worked, I would ask what is the relationship between the value of b, 6, and the period being 60? If you didn't have the sliders, how could you have figured that out?
- Why does it make sense that the period is 60 when b is 6? What would you expect the period to be if b was 9? Why?
- I noticed that as you were looking for a value of b that worked, that you kept making b larger. How did you know to make it bigger? How is the value of b related to the period?

In this second video clip, Rhian and Shakira are still working on Introduction to Sine Desmos Task. They are now working on page 14 of the task, where they are expected to determine the function equation to match a given graph. You are going to join their discussion as they are trying to figure out the value of b to determine the period. Watch the video and listen carefully to what the students are saying and the ways they are interacting with the technology.

Rhian and Shakira Exploring Period of the Sine Function (Clip 2)

Q5. Describe the issue(s) or challenge(s) that the students are facing while determining the period. Provide evidence from the clip to justify your answer.

- They are struggling to see the relationship between b and the period. We see it at the very beginning when they try 360 divided by 60 and get something that surprises them.
- The challenge they are facing is how to connect what they know about period being the difference between the two "peaks" with how do I find a b that gives me the distance I need. The evidence is when the guess that b should be bigger and pick 60 to try and see the difference between the peaks is really small, not what they expected to see.
- The issue is that they understand period as the distance between two consecutive max points, but they don't know the relationship between the period, b, and 360 yet. See what happens when they try $b = 60$ at the 0:40 mark in the video.

Q6. Attend to how the pair use their prior knowledge (shown in the previous video clip) and the technology to deepen their understanding of period and address the issue(s)/challenge(s) you described in Q5.

- The students used their knowledge of period as the distance between peaks and what they had done on a previous page to make sense of this situation. They first

tried some values for the b slider based on the difference idea (they tried 60, 12, 2, and -2) and each time saw it did not work. The 60 and 12 seemed to surprise them. Then they went back to how they figured it out on the previous example like this one. Shakira explained that for that one they did 120 times what is 360 or 360/120 was b. So they thought they would need to use the 720 and the 360 in some way to either multiply or divide. Eventually Shakira shared that she set up this one just like the last one so it would be 360/720 which Rhian agreed with and said based on that it should be ½. They tried it and it worked. Then they made sense of the reflection across the midline as needing a negative ½.

- They start by using their understanding of period as the difference between peaks to write their function to match the graph. They anticipate that b needs to be "further away from zero" because the graph given is wider than the base function. Then they say it is 360 divided by 6 or 60 (it is not clear why they say360 or 60) and try 60 for b. They see a very "tight" sine wave and say "woah" indicating that the graph surprises them. Next they look for the difference between the peaks of the given graph (their understanding of period from earlier) and determine it is 720. Shakira says it should be 720 divided by 60, which is 12 and tries 12 for b. Again, the see a graph that has a smaller period than expected. Rhian then asks why they are dividing by 60. Shakira explains that she thinks it is "that" divided by 360 would give you b which is 3 (she went back to page 12 to explain (prompted by Ms. Fye). She more clearly states "360 divided by a number will give you 3", in this case the period is "the number" and 3 is the value of b. Ms. Fye then asks how we can use that idea with the 720 on this page. The students go back to page 14 and also have white boards that they start to write things on and test some values. Rhian notes that 720 would be 2 times 360, but she doesn't feel like that is helpful. She does try it, setting b to 2. They observe that it does not work. Shakira clarifies with Ms. Fye that 360 is the period of the base function. Both girls record on their whiteboards what they did on the last problem 360/120 = 3. Shakira writes (and says) 720/360 = 2. Shakira sets a to 3 since she knows the amplitude is 3 so the graph is closer to being correct. As Rhian is trying values for b on the graph, we see Shakira write down 360/720. Rhian says she thinks it should be -2 but isn't sure how to express why. When Ms. Fye prompts her, Shakira says that she put 360 on top of the fraction because she did that for the other example too, so she though it should be the same here. Rhian agrees and says based on that it would be 0.5. They try b as 1/2 and then make it negative to reflect the graph. They like what they got for b. Ms. Fye pushes back on the negative and asks them to take it out and test of the period is still the same. They decide it is using their method of differences. They then explain that the negative reflected it across the midline.
- To address their challenge here they looked back and a previous example and looked for a pattern. In the previous one they did 360 divided by the difference they found on the graph to find the value of b. Though at first they described it as multiplying (120 times something would give us 360). They both worked on whiteboards to rearrange the numbers 360 and 720 a few different ways using multiplication and division. Seeing that 360 times 2 is 720 they tried $b = 2$ and then $b = -2$. After Ms. Fye asking Shakira to talk about using 360 in the

numerator on the first example should mean using it in the numerator here too they both thought that made sense and Rhian said then it should be .5. They tried it and seemed think it was correct, but Rhian quickly changed it to -½ so that the graph would reflect. Ms. Fye guided them in unpacking which value was related to the period (½) and why adding the negative helped match the graph. She did this by having them identify the parts on the graph and seeing if the period was the same in the red and blue graphs without the negative first.

Q7. At the conclusion of this video clip, what is Rhian and Shakira's understanding of period? Provide evidence from the clip to justify your answer.

- At the end of this clip Rhian and Shakira seem to understand period as both the distance between the peaks of the graph (as evidenced by their finding the difference between them many times in the video) and they know they can find the value of b by dividing 360 by the period (we see this when they see the pattern and can use it to find the value of b toward the end). It is not clear that they understand that b is how many times the function cycles in 360 degrees.
- They understand that to find b they divide 360 by the distance between the peaks (the period). The evidence is the last few minutes of the clip when they see the pattern and find b for this example.
- I'm not sure what their understanding is when I look at the definition. There is further evidence in this clip that they understand period as the distance from one peak to the next (the horizontal length). Even though it seems like they have figured out that to find the value of b they take 360 and divide by the period (the procedure), I am not convinced that they understand that b describes how many times the function cycles over 360 degrees.

Q8. One of the performance goals for this task was to generalize how to find the period given a graph of the sine function. What would you, as the teacher, do next to support their mastery of this goal? Include any assessing or advancing questions you might pose and explain why you would or would not use technology in your next steps.

- I'd like to see them generalize what they seem to have figured out. I think I would pose a few more examples for them (given this graph, what is the equation) just to make sure they are using what they seem to have figured out consistently. Then I would ask them how they would describe to a friend how to find the value of b no matter what the graph is they are given. I would use the technology so they had a way to observe if they were correct or not in their assumptions as they worked.
- I would set up a few more pages in Desmos like pages 12 and 14 where they were given a graph and had to determine the function. I'd use the technology so they had a way to test their conjectures as I am not sure they are sure about the pattern for finding b yet. After a few more of those I'd ask if they can write a

sentence or an equation that describes how to find b for the equation given any graph.

- I think they still need a way to check themselves (they figured that one out, but still don't seem confident), so I would use technology still. I'd give them graphs like they had on this example and ask for the equation or maybe just for the b value. I think 3 more examples would be good. I'd want some with periods larger than 360 and some that are smaller than 360. Also some where the peaks are not on pretty numbers. After more examples I would ask them if they can generalize what they found.