Module 6 Overview Document

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	Homework	6.1 Comparing and Contrasting Function Parameter Explorations Quadratic Functions – Vertex Form
	Day 1	20 min	6.1 Discussion
		55 min	6.2 Matching Performance Goals to Task Design: Quadratic Function Parameter Explorations
		Homework (Optional)	6.3 Adapting an Existing Desmos Quadratic Parameter Task

6.2 Facilitation Notes

We recommend assigning Bailey et al. (2021) as a reading AFTER teachers complete this entire task. The practitioner article discusses the affordances and constraints of each of these three Desmos tasks and provides insight into various performance goals that may be used with each of the tasks.

Depending on your context, it might be helpful for your teachers to review how to write good performance goals prior to starting 6.2. The following two articles provide thoughts for discussions on writing performance goals that align to standards and on choosing tasks based on performance goals.

Lo, J. J., & White, N. (2020). Selecting GeoGebra applets for learning goals. *Mathematics Teacher: Learning and Teaching PK-12, 113*(2), 156–159.

Hunt, J., & Stein, M. K. (2020). Constructing goals for student learning through conversation. *Mathematics Teacher: Learning and Teaching PK-12, 113*(11), 904–909.

After completing your discussion on 6.1, switch the makeup of your teacher groups and have them work on 6.2. While they work, walk around the classroom and ensure discussions include evidence about how the students engage with the different Desmos tasks and how their engagement led to their understanding.

Depending on timing and your own objectives for this lesson, you may choose to have the whole class work together on Q4 while revisiting the various performance goals the teachers wrote on 6.1.

After this task, you are ready to assign the MTLT article about this task:

Bailey, N. G., Yalman Ozen, D., Lovett, J. N., McCulloch, A. W., & Cayton, C. (2021). Parameters, sliders, marble slides, oh my! *Mathematics Teacher: Learning and Teaching PK-12, 114*(5), 386–394.

- Video 1: Amya and Erica Engage with the Multiple Graphs Activity
- Video 2: Sample Directions Parameter h Slider
- Video 3: Sara and Julian Engage with the Quadratic Sliders Activity
- Video 4: Directions for the Quadratic Marbleslides Activity
- Video 5: Kaevel and Nasiyah Engage with the Quadratic Marbleslides Activity
- Video 6: Eden and Ethen Engage with the Quadratic Marbleslides Activity

Note: At this point, you may choose to have your teachers work on the optional assignment 6.3 to provide them with practice adapting activities to better align with specific performance goals. The video clips in the article have been included in this Module so you can access them easily with your teachers.

6.2 Samples Responses
Matching Performance Goals to Task Design: Quadratic Function Parameter Explorations

You have examined three different Desmos Tasks related to exploring the parameters of the quadratic function in vertex form, $f(x) = a(x-h)^2 + k$ and considered the performance goals that each task addresses. Now you are going to watch video clips of pairs of high school students engaging with each task. As you watch, focus on the ways in which the students are engaging with the tools in the task and evidence that they are making progress toward the performance goals you anticipated.

Q1. In the following video clips, you will see two pairs of students working on the *Quadratic Marbleslides Task*. Watch the videos (as many times as needed) and then answer the following questions:

- The Marbleslides Task
- Eden and Ethen
- Kaevel and Nasiyah

a. How are Eden and Ethen (who have prior experience with parameters of quadratics in vertex form) & Kaevel and Nasiyah (who have no prior experience) engaging with the task? (e.g., What values are they trying? To which representations are they paying attention?)

Sample teacher responses for Eden and Ethen:

- Eden and Ethen were trying the a value that affects the opening of the parabola and the width of the parabola. They already had prior knowledge that the larger the value of a the more narrow the opening and the smaller the value of a the wider the opening will be.
- Eden and Ethen knew that larger values of a made the graph narrower, so they knew right away to try decimal numbers in order to collect the stars in a wide quadratic pattern. They plugged in a small decimal and watched the graph change. [While .2 is not the actual value for I ($a = .25$), the exact value didn't necessarily matter to show their understanding of the effect of a. "The bigger the number, the more narrow it is" illustrates their understanding of parameter a as does Eden's question about "how narrow it can be". This shows her curiosity about the different graphs that can be created].
- Eden and Ethen looked at the stars as comprising a quadratic graph (invisibly connecting the stars in their head) and thought about a possible transformation of the parent function. They immediately knew to try a decimal value <1 as they knew the stars comprised a quadratic wider than the parent quadratic function.

Sample teacher responses for Kaevel and Nasiyah:

- Kaevel and Nasiyah did trial and error by trying to slide the graph around (only left, right, up and down). They focused on moving the graph without looking at the equation itself.
- Kaevel and Nasiyah engaged with the activity through a guess and check method. Through trying numbers that they thought might be close they began to get the graph closer to the desired look. They relied entirely on the picture of the graph after trying a number to see what needed to change.
- Kaevel and Nasiyah moved the sliders without understanding the effect of each parameter. It is unclear whether or not they know there are parameters in the equation that are changing. They seem to know that the value they see on the axes has some effect on the graph, but do not know how the two are related. Overall they seem to be playing a "matching game".

b. Interpret how the students' engagement with the task influenced their current understanding.

Sample Teacher Responses for Eden and Ethen:

- Eden and Ethen during the activity were reinforcing what they have already learned about quadratic parameters (e.g. the bigger the number, the more narrow it is) and further developed their understanding by testing a value to

see how narrow the quadratic could be causing them to have a deeper understanding of the concept.
- Eden & Ethan had prior experience with parameters so they knew the a in their function needed to be changed. Eden then asked, "how narrow can it be?" so she tried $a = 10$. They understood from the beginning that the bigger the number for a, the narrower the graph gets.
- Eden and Ethen's engagement helped reinforce their understanding through them testing further values to determine narrowness.

Sample Teacher Responses for Kaevel and Nasiyah:
- While Kaevel and Nasiyah engaged with the activity by guessing and checking the questions till it worked and never really made a connection with what the parameters do.
- Kaevel and Nasiyah seemed to get frustrated at the end when they couldn't quite figure out what the different parameters do. At least one of them seemed like they were on the verge of giving up because they were discouraged by not seeing how to make the parabola flip downwards.
- Kaeval and Nasiyah don't have prior knowledge when it comes to this topic and I do not see them learning much from what they are doing. It seems more like they are just trying values and not seeing why they do/do not work. I do not think this is influencing their current understanding of the content because they are mostly just doing guess and check the whole time, when that is not what we want students to do past these activities.

Q2. In the following video clip, you will see Sara and Julian working on the *Quadratic Sliders Task*. Watch the video (as many times as needed) and then answer the following questions:

The Quadratic Sliders Task

Sara and Julian

a. How are Sara and Julian engaging with the task? (e.g., What values are they trying? To which representations are they paying attention?)

Sample Teacher Responses to this question include:
- Sara and Julian considered the effects of one parameter at a time and focused on how the location of the graph changed with respect to the coordinate plane. For h and k, they moved the sliders to whole numbers and then looked at the x- and y-coordinates of the vertex (they did not have the language for vertex; instead they talked about changing the x or changing the y). When they considered parameter a, they noticed the wideness of the graph changed and were surprised to see the graph flip. They did not make a conjecture as to why this occurred.

- They have a good understanding of how each parameter changes the location and shape of the graph, by moving it left and right with h, up and down with k, and widening the curve with a.
- Sara and Julian seem to mostly be focusing on the graphical representation as they slide the sliders. They observe that sliding h moves the graph left to right, sliding k moves it up and down, and sliding a changes how wide/narrow it is, as well as its "direction."

b. Interpret how the students' engagement with the task influenced their current understanding.

Sample Teacher Responses to this question include:

- Their careful engagement with each slider one at-a-time certainly helped to introduce the parameters for a quadratic function because they were able to observe a correlation between the value of the slider and position/manipulation of the graph. - For each parameter, it seems they were able to quickly recognize how each of the parameters influenced the graph (h: moves along x-axis, k: moves along y-axis and a: gets bigger or smaller). They do not have the formal language for the vertex, but are developing an understanding of each parameter.
- Over time, they were starting to understand the idea of focusing on the vertex, and that the h changes position on one axis, while the k changes the other. The sliders also had an effect on how they approached the problem, and subsequently their understanding of vertex form, since they were "playing" in a space that allowed them a more guided experience with one parameter at a time. They were still developing an understanding of the parameter, a.
- I believe that the activity helped their understanding of the topic. They seem to know what the parameters themselves do in simple terms in relation to the graph. They would need more understanding of how it works and why it works the way it does, but I think this is a good introduction to the topic for the students to gain a foundational knowledge.

Q3. In the following video clip, you will see Amya and Erica working on the *Quadratic Parameters Task*. Watch the video (as many times as needed) and then answer the following questions:

The Quadratic Parameters Task

Amya and Erica

a. How are Amya and Erica engaging with the task? (e.g., What values are they trying? To which representations are they paying attention?)

Sample Teacher Responses to this question include:

- From the start of the video, we see that Amya and Erica are testing various whole numbers for the parameter h. They have tested both positive and negative values and explore the difference between $x - -5$ and $x + 5$, and appear to see that they are the same which leads them to determine that a minus moves the graph right and a positive left...this is indicated by their discussion of it "moving to the other side." For k, they determine that a positive moves it up, but don't seem to know if the negative goes down or affects the wideness. When exploring parameter, a, they test larger whole numbers to see how much "thinner" the graph can get. Their exploration of negative values for a leads them to determine that a and $-a$ are the same thing but flipped over.
- Amya and Erica test both positive and negative whole numbers to determine the effects of the parameters. They seem to be comparing all of their graphs against each other, not just comparing between the mother function and their newest created graphs.
- Amya and Erica are cumulatively testing the effects of different whole numbers (positive and negative) for each of the three parameters. They determine that h results in a horizontal shift (moving to the other side of the sign) and that k results in a vertical shift. They notice that a results in a vertical stretch or compress between "smaller or bigger" values, but they determine this comparing their various graphs to each other and not to the parent function. They notice that negative values of a flip the graph over.

b. Interpret how the students' engagement with the task influenced their current understanding.

Sample Teacher Responses to this question include:

- Amya & Erica's engagement with this activity helped them notice some key aspects of this topic. It seemed like they had no exposure prior to this, but they recognized that h moved the parabola left & right, k moved the parabola up and down, and a made the parabola wider or narrower or flipped it over the x-axis. Those are great beginning steps that can lead to some discussion and more formal understanding down the road.
- Amya and Erica are engaging in this activity by creating new functions and because of this they are watching and understanding what they have created and how they look different with every change. They are constantly asking questions and testing different values as they develop an early understanding of the effects of each of the parameters. Because they do not always check values <1, they have not yet seen all of the effects of the parameter a.
- Amya and Erica were testing a bunch of numbers to develop their understanding of the properties of each parameter. They understand that h moves the graph "along the x-axis" and k moves the graph up and down. One thing that sticks out is testing the negative values for a in the last clip, the students not only recognized the compressing and widening

of the graph, but also that it can reflect the graph and make the parabola open downwards.

Q4. Now that you have watched students engaging with each task, revisit the performance goals that you wrote for each in 6.1 (Comparing and Contrasting Function Parameter Explorations Quadratic Functions – Vertex Form). Using evidence from Q1–Q3, refine your work. Record two performance goals that you believe each activity could address.

a. Performance goals *Quadratic Marbleslides* could address:

Note: These are the same sample teacher responses as those listed in 6.1 Q2

- Students will review the effects of various parameter changes to quadratic functions in vertex form
- Students will expand on their previous knowledge of quadratic functions to practice adapting functions to fit points through parameter changes to quadratic functions in vertex form
- Students will be able to create a quadratic equation that matches up with given points on a graph.
- Students will be able to notice differences between quadratic functions, and their graphs.
- Students will be able to input values so that the graph of the corresponding function goes through specific points.
- Students will be able to recognize that not only positive values can be inputted into the functions.
- Students will be able to identify what changes occurred to the parameters of the parent quadratic function to obtain the graphs they create on the screen
- Students will be able to determine a graph to match the given points and identify the corresponding function.
- Students will self-monitor their knowledge about parameter changes to quadratic functions in vertex form

b. Performance goals Quadratic Parameters could address:

Note: These are the same sample teacher responses as those listed in 6.1 Q2

- Students will individually observe different parameter changes to quadratic functions in vertex form
- Students will record various parameter changes to quadratic functions in vertex form
- Students will compare various changes to parent quadratic function in vertex form.
- Students will make and test conjectures about the effects of the individual parameter changes to quadratic functions in vertex form
- Students will be able to write the equation of a quadratic in vertex form and understand how the different parameters affect the graph.

- Students will be able to identify what changes occurred to the parameters of the parent quadratic function to obtain the graphs they create on the screen
- Students will be able to match a quadratic function in vertex form to its graph
- Students will be able to recognize what parameter changes in a quadratic graph correspond to each parameter in vertex form.
- Students will be able to graph quadratic functions when given a function in vertex form.
- Students will be able to explain how the quadratic functions they create are different from the parent function.
- Students will be able to define the effects of parameters a, h, and k.

c. Performance goals Quadratic Sliders could address:

Note: These are the same sample teacher responses as those listed in 6.1 Q2

- Students will be able to identify the a, h, and k values by dragging the slider.
- Students will explore the effects of parameter changes to quadratic functions in vertex form
- Students will test conjectures about the effects of individual parameter changes to quadratic functions in vertex form
- Students will compare how moving different sliders, changing the values, affect the graph in different ways.
- Students will be able to identify the difference between the positive and negative changes of the various parameters in vertex form and how they affect the graph.
- Students will be able to match a quadratic function in vertex form to its graph
- Students will be able to recognize what parameter changes in a quadratic graph correspond to each parameter in vertex form.
- Students will be able to graph quadratic functions when given a function in vertex form.
- Students will be able to describe the relationship between parameters a, h, and k and the graph of the function.
- Students will be able to explain how various quadratic functions are different from the parent function.