Module 5 Overview Document

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	5 min	5.1 Vending Machine Task (Revisiting the Concept of Function Version)
	Day 1	30 min	5.1 Discussion
		45 min	5.2 Noticing Student Thinking: Machine G of the Vending Machine Task
		Homework	5.3 Noticing and Predicting: The Vending Machine Task*
	Day 2	15 min	5.3 Discussion
		60 min	5.4 Selecting & Sequencing: Defining Function with the Vending Machine Task*
		Optional Homework	5.5 Anticipating Student Thinking: Concept Images of Function and the Vending Machine Task OR 5.6 Orchestrating Whole Class Discussions: Defining Function with the Vending Machine Task*

** Indicates there are two versions of this task—one in which secondary school students are being introduced to the function concept for the first time and one in which secondary school students are revisiting the function concept.*

5.3 Facilitation Notes

We suggest sharing the task worksheet electronically as it has links to associated documents and videos embedded within.

There are two versions of this task you can use to differentiate if you choose. The only difference is the student videos.

The Facilitation Notes for 5.3-R Noticing and Predicting: The Vending Machine Task (Revisiting the Concept of Function Version) are on pages 1–12: Sample Responses (pp. 1–9) and Additional Task Commentary (pp. 10–12).

The Facilitation Notes for 5.3-I Noticing and Predicting: The Vending Machine Task (Introduction to the Concept of Function Version) are on pages 13–21: Sample Responses (pp. 13–18) and Additional Task Commentary (pp. 19–21).

5.3-R Sample Responses
Noticing and Predicting: The Vending Machine Task (Revisiting the Concept of Function)

The learning goal for the Vending Machine Task is:

- Students will understand that a function is a special relationship between an input and output (independent variable and dependent variable) in which each input is mapped consistently to an output.

Specific performance goals include:

- In the context of a vending machine simulation, students will determine if a mapping is a function.
- Students will describe what a function is in their own words.

Watch the videos C and D Student Responses, of two groups (Angel & Alicia and McKenzie & Tia) engaging with the high school student version of the

Vending Machine Applet

While watching each video, focus on the students' language. Based on their responses, predict the students' responses for machines E and I in the student version of the applet. Provide evidence from the video to justify your responses.

Mapping for machines:

Table 2: This table provides the mapping for Machines C and D.

C	Red – Blue Blue – Silver Silver – Green Green – Red
D	Red – Random Pair Blue – Blue Silver – Silver Green – Green

Machines C and D Student Responses - Group 1

Angel & Alisa Explore Machines C and D

Q1. Attend to how Angel and Alisa engaged with the applet to decide which machine was or was not a function.

An ideal answer should include technological actions that Angel and Alisa use to explore Machines C & D, as well as their conclusion about each machine based on these actions. Focus on what they did and said. One example might be:

- Angel and Alisa test each can in Machine C, then test each can in Machine D to look at the output. They are careful to 'take can' each time in the first round. In the first round they notice that Machine D gives two cans for 'red cola'. In round two of their testing they switch users and the second test for Machine C, they do not always 'take can'. They do say Machine C is a function because you may get a different color can than what you press, but you get the same can each time you press. For Machine D they use a combination of the fact 'red cola' produces two cans and that the color is not the same each time to conclude it is not a function.

Q2. Interpret Angel and Alisa's understanding of function based on their engagement with the applet. Use examples from the screencast as evidence to show how you know what they do or do not fully understand.

An ideal answer should include a coordination of how their engagement with the app led to their conclusion of consistent, single output. One example might be:

- Angel and Alisa interacted with each button of the applet multiple times to examine outputs. For Machine C they concluded that it was a function because even though you do not get the color you select, a single can is produced and is a consistent color each time. For Machine D they conclude it is not a function since red produces two cans, and the cans are not the same color each time. Their statement that a function has a consistent, single output is directly related to their interaction with the applet. I do wonder if they had explored a machine that gave two cans that were consistently the same color, what they would have said. Would they have just focused on the fact it was two cans and ignored the consistent part? Or grappled with the idea of getting two consistent cans, in other words a consistent output each time? For now they understand that a function should have a consistent, single output in order to be a function.

Mapping for machines:

Table 3: This table provides the mapping for Machines E and I.

E	Red – Red Blue – Blue & Random Silver – Silver Green – Green
I	Red – 2 Silvers Blue – Green Silver – Red Green – Blue

Q3. Create a scripted conversation between Angel and Alisa that describes how you

predict they will engage with Machines E and I to determine whether or not each is a function. For your script, denote engagement with the applet using parenthesis. See example below. Briefly justify your predicted script including evidence from the video.

Example:
Student 1: (clicks on the red cola button) It puts out green.
Student 2: Wow.
Student 1: (clicks on red cola button again). It still puts out green.
...

An ideal script should mimic the actions Angel and Alisa took when engaging with Machine C & D. Their ideas focused on producing two cans and producing a consistent outcome each time, so the script should address how they use these ideas with Machines E & I. Also how they might deal with the fact that Machine I produces two, consistent cans each time. Potential examples may include:

Table 4: This table can be used to write your scripted conversation for Angel and Alisa. The first column lists the machines by letter. The second column is empty for your predictions. The third column is for your script with justification.

Machine	Prediction (Function or Non-Function)	Script with Justification Your script should include 1) precise language they will use when talking to each other; 2) exactly how the students will engage with the applet; and 3) provide insight into how their understanding of function will lead them to this conclusion.
E	Non-Function: It gives out two cans and one of them is a different color each time.	Student 1: (clicks on red cola button) Ok. (takes can) Student 2: It gave red. Student 1: (clicks on blue cola button) Oohh! Two cans. (takes can) Student 2: Oohh! But not the same color as each other like the last machine. Student 1: (clicks on silver cola button). Silver. (takes can) Student 2: Silver Student 1: (clicks on green cola button). Green. (takes can) Student 2: Green. *Pair cycles through clicking and taking can each time, observing. Same results for red, silver, and green. Blue produces two cans - first can is blue like before, second can is not the same as before but different than blue.* Student 1: Ok, so this one acts like Machine D where three buttons give the color you select each time, but one button gives two cans that are not the same each time. Student 2: Yeah, they are also not the same color as each other. The first can is blue, but the second can changes. The other machine the pair was the same color as each other but changed.

		Student 1: I still don't think it is a function because it is not the same each time. Student 2: Yep. Not a function.
I	Function: It gives out different color cans for each can, but it gives out the same color for each button each time. Red gives two cans, but the cans are the same color as each other and the same silver each time.	Student 1: (clicks on red cola button) Hmm. 2 silver cans. (takes can) Student 2: Two cans. Student 1: The other machines with two cans were not a function. Student 2: Yeah, but let's see what the other buttons do to be sure. Student 1: (clicks on blue cola button) Green. (takes can) Student 2: Green. Student 1: (clicks on silver cola button). Red (takes can) Student 2: Red Student 1: (clicks on green cola button). Blue. (takes can) Student 2: Blue. *Pair cycles through clicking and taking can each time, observing. Same results for blue, silver, and green. Red produces two cans - both silver.* Student 1: Ok. So this one is like Machine C and D combined. The buttons don't give the color you select, but they do give the same color for each button each time. Student 2: But red gives two silver cans each time. Student 1: Yeah, but since it gives the same cans and color each time, I think it is a function. Student 2: I guess, maybe. The other machines that gave two cans did not give the same pair of cans. It changed each time. This machine it stays the same. Student 1: Function. Student 2: Function.
I	Non-function: It gives out two cans.	Student 1: (clicks on red cola button) Hmm. 2 silver cans. (takes can) Student 2: Two cans. Student 1: The other machines with two cans were not a function. Student 2: Yeah, but let's see what the other buttons do to be sure. Student 1: (clicks on blue cola button) Green. (takes can) Student 2: Green.

		Student 1: (clicks on silver cola button). Red (takes can) Student 2: Red Student 1: (clicks on green cola button). Blue. (takes can) Student 2: Blue. *Pair cycles through clicking and taking can each time, observing. Same results for blue, silver, and green. Red produces two cans - both silver.* Student 1: Ok. So this one is like Machine C and D combined. The buttons don't give the color you select, but they do give the same color for each button each time. Student 2: But red gives two silver cans each time. Student 1: Yeah, two cans means two outputs. I don't think a function can have two outputs. Student 2: No, it can't. Student 1: So, this one is not a function. Student 2: Nope, not a function.

Mapping for machines:

Table 5: This table provides the mapping for Machines C and D.

C	Red – Blue Blue – Silver Silver – Green Green – Red
D	Red – Random Pair Blue – Blue Silver – Silver Green – Green

Machines C and D Student Responses - Group 2

McKenzie & Tia Explore Machines C and D

Q4. Attend to how McKenzie and Tia engaged with the applet to decide which machine was or was not a function.

An ideal answer should include technological actions that McKenzie and Tia use to explore Machines C & D, as well as their conclusion about each machine based on these actions. Focus on what they did and said. One example might be:

- For Machine C, McKenzie and Tia click on each can to see the output, then take can before choosing the next can. They cycle through in order red, blue, silver, green twice. Then a suggestion is made to click in a different order and not take can. They notice that the same output occurs even when they click in a different order and do not take can. They conclude it is a function, and write on their paper

"reliable". When they move on to Machine D, they click on red and see two silver cans. They immediately say Machine D is not a function without stating any reasoning. The teacher asks them why D is not a function, and one young lady responds because there are two outputs. During this time they are still clicking on cans to observe outputs, and the second young lady says because there are two outputs. They write on their paper, "2 outputs for same input".

Q5. Interpret McKenzie and Tia's understanding of function based on their engagement with the applet. Use examples from the screencast as evidence to show how you know what they do or do not fully understand.

An ideal answer includes a coordination of how their engagement with the app led to their conclusion that a function has a reliable, single output. One example might be:

- McKenzie and Tia cycle through the cans in different orders and play around with taking the can and not taking the can for the first machine. Once they find that they get a 'reliable' output they determine it is a function. That makes me think they understand that a function should have a reliable output in order to be a function. Once they explore Machine D and see that they get two cans, not of the same color each time. They conclude that it is not a function because it produces two cans, which makes me think that they understand a function should produce a reliable, single output each time. The fact that they did not think deeply about the reliability of the color of the two cans makes me wonder what they might do if a machine produces two cans, reliably of the same color/color combo.

Mapping for machines:

Table 6: This table provides the mapping for Machines E and I.

E	Red – Red Blue – Blue & Random Silver – Silver Green – Green
I	Red – 2 Silvers Blue – Green Silver – Red Green – Blue

Q6. Create a scripted conversation between McKenzie and Tia that describes how you predict they will engage with Machines E and I to determine whether or not each is a function. For your script, denote engagement with the applet using parenthesis. See example below. Briefly justify your predicted script including evidence from the video.

Example:
Student 1: (clicks on the red cola button) It puts out green.
Student 2: Wow.
Student 1: (clicks on red cola button again). It still puts out green.
...

An ideal solution would focus on the girls' conclusion from Machines C & D that two cans (or outputs) indicates it is not a function. They did not talk with each other or express their reasoning/thinking out loud as much as the first pair, so the interaction should reflect that.

Table 7: This table can be used to write your scripted conversation for McKenzie and Tia. The first column lists the machines by letter. The second column is empty for your predictions. The third column is for your script with justification.

Machine	Prediction (Function or Non-Function)	Script with Justification Your script should include 1) precise language they will use when talking to each other; 2) exactly how the students will engage with the applet; and 3) provide insight into how their understanding of function will lead them to this conclusion.
E	Non-Function: It gives out two cans and one of them is a different color each time.	Student 1: (clicks on red cola button; sees red can) Ok. (takes can) Student 2: (Looks at screen and nods) Student 1: (clicks on blue cola button) Two cans. (takes can) Student 2: Two cans so not a function. Student 1: Uh huh. Let's test the others to be sure (clicks on silver cola button). Silver. (takes can) Student 2: (Looks a screen and nods) Student 1: (clicks on green cola button). Green. (takes can) Student 2: Ok, so not a function since blue gives two cans.
I	Non-function: It gives out two cans.	Student 1: (clicks on red cola button) 2 silver cans. (takes can) Student 2: Two cans, so not a function. Student 1: The other machines with two cans were not a function, so yeah. Student 2: Let's see what the other buttons do to be sure. Student 1: (clicks on each button and observes color given, takes can each time) Student 2: (watches screen and nods as student 1 is clicking through) *Pair cycles through clicking and taking can each time, observing. Same results for blue, silver, and green. Red produces two cans - both silver.* Student 1: Ok. So not a function because red gives two silver cans.

Q7. How has your own understanding of function been influenced by thinking about how high school students develop the concept of function?

From the perspective of a teacher and ideal solution would include a focus on consistent/reliable/predictable outcome and deal with the idea of producing two cans/outcomes that are not consistent vs. that are consistent. Some examples may include:

- The vending machine applet pushed me to think about what it means to be a function in terms of what is selected/input and what results/output. When you get a consistent, single output from the same input then you are dealing with a function. The vending machine applet helps high school students experience this in a familiar but non-math way.
- The set-up of the vending machines each having one button that produces two cans presents a unique opportunity for high school students to think about getting the same output each time you choose a given input. Each pair came to understand that two outputs are not allowed for it to be a function, which is good.
- I think the fact that the use of one machine that gives a single, consistent output each time sets students up to easily see that getting two cans/outputs each time is not a function. What I am wondering is about the machine that produces the same pair of cans each time. Machine I gives two silver cans consistently each time, so I wonder if high school students would rely on the fact that it is a consistent outcome each time, so it would be a function. Or if they would say it is not a function just because it produces two cans. The students in this assignment went the path of two cans means not a function, but I could see a student arguing for the consistent output of two cans being a function.

5.3-R Commentary
Noticing and Predicting: The Vending Machine Task (Revisiting the Concept of Function Version)

As you are preparing to begin a whole class discussion on the Vending Machine Task, teachers should have their worksheet with answers available and their laptops with the applet open to reference. In addition, we recommend passing back out the definitions of function teachers wrote prior to engaging with the task.

There are two parts to this discussion, the first is focused on discussing the specific machines in the applet; the second is focused on developing a definition of function. Furthermore, teachers will move in and out of small group and whole class discussion. Having your space set up for small groups that can stay arranged that way and still have a whole class discussion will be beneficial.

Introduction

In this assignment teachers will watch short video clips of pairs of HS students as they discuss Machines C and D from the Vending Machine Task. As a reminder, those machines behave as shown below.

Table 1: Machine C and D behaviors

Machine C	Machine D
Red Cola → Blue	Red Cola → Random Pair
Diet Blue → Silver	Diet Blue → Blue
Silver Mist → Green	Silver Mist → Silver
Green Dew → Red	Green Dew → Green

We have included student names in all videos to make it easier for teachers to talk about the students' strategies. We have found that using names provides a sense of familiarity with the students and helps to mitigate the use of deficit language in discussing the students' work. Even so, it is important to remind teachers about honoring the power of the students' rough draft ideas as they discuss their current strategies and understandings.

Given that this assignment specifically asks teachers to attend to student strategies and interpret their understanding (the first two components noticing students' mathematical thinking), it might be helpful to remind them of the Noticing Students' Mathematical Thinking in Technology-Mediated Learning Environments Framework (see Module 0 for details).

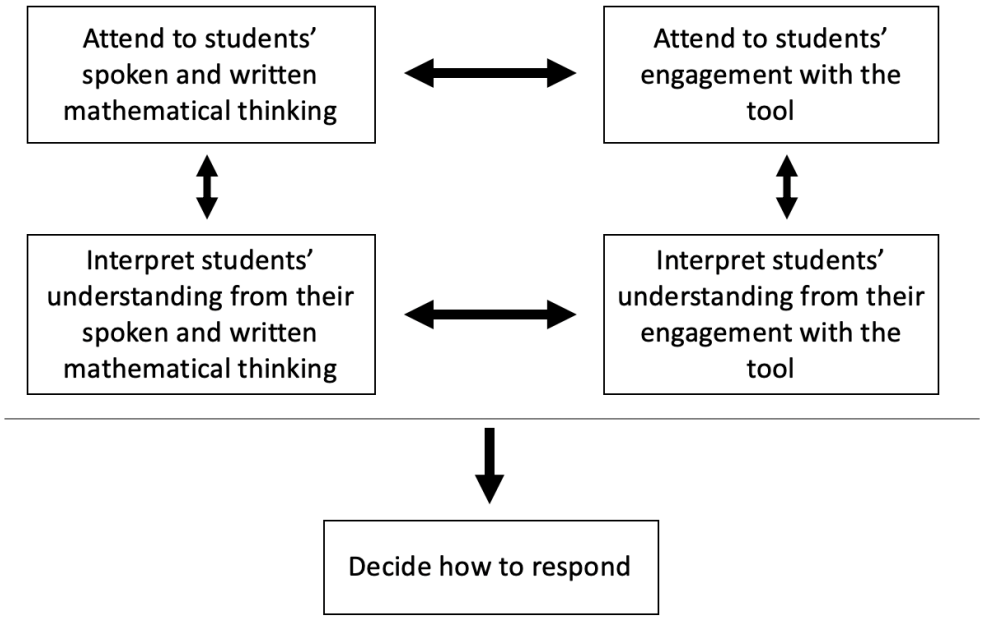

Figure 1. Noticing Students' Mathematical Thinking in Technology-Mediated Learning Environments Framework

Teacher Independent Work

Teachers should complete this assignment for HW, working independently, and then come prepared to share and discuss their responses the next class session. We recommend this being completed outside of class as teachers need time to watch the videos (possibly more than once) and reflect on them as they craft their responses to the prompts.

Whole Class Discussion

Begin the whole class discussion by displaying the framework for noticing students' mathematical thinking in a technology-mediated learning environment. Ask teachers what the different components *generally* might look like in the context of the Vending Machine Task. In other words, what sorts of things should a teacher watch for/listen for when noticing student work with this task? Record these on the board as they are generated.

Responses might include:

- The way students test the individual buttons (i.e., clicking just once or more than once, using the take can button between clicks or not)
- Whether students are talking about the color of the cans, the number of cans, the consistency in the outputs when a particular input is pressed
- The personal definitions of function, input, or output they appear to be using as they test the cans
- Particular phrases they use to explain their thinking

Next, pick a few of the ideas listed (or combinations of them) and ask teachers what insight they provide to students' current (rough draft) understanding of function. For example, you might ask "What if you noticed a pair of students only testing the buttons on the machine once and recording the number of cans they see in the output. What interpretations might you make about their understanding of function?" (Avoid asking about the particular things they will be discussing related to the two groups of students they observed in the assignment.)

After this general discussion, move to talking specifically about the two groups of students they observed in the assignment—Group 1: Angel and Alisa and Group 2: McKenzie and Tia. It is helpful to record what teachers share for their attend (#1 and #4) responses, so everyone can look back at them when discussing their interpretations and predictions.

As teachers share their interpretations, make sure they provide justification from what students said or wrote AND their engagement with the machines. This coordination needs to be emphasized as an important feature of noticing (and monitoring) students' work on technology-enhanced mathematical tasks.

When you get to discussing teachers predictions, it is helpful to first poll whether or not they predicted the students would classify each machine as a function or not. This can be done informally or using a polling/survey technology. But a quick glance to know if there is agreement or not among their predictions will help you orchestrate this part of the discussion.

Possible discussion prompts include:

- (If there is not agreement) Using evidence from the video, explain your reasoning behind predicting students' X and X will decide machine X is (or is not) a function.
- You were asked to create a script for your prediction. How did you go about creating that script? What did you draw upon from the videos to do this?
- Why do you think I asked you to predict their responses to Machines E and I specifically?
- Are there any other machines you feel as if you can predict their responses to based on what you saw in the video? Why or why not?
- Are there specific ways in which the students interacted with each other that were important to your prediction? Why?
- In what ways might making predictions like this (both the mathematical and interactional aspects of the predictions) be helpful to you if you were monitoring a whole class of students working on this task?

5.3-I Sample Responses
Noticing and Predicting: Vending Machine Task (Introduction to the Concept of Function Version)

The learning goal for the Vending Machine Task is:

- Students will understand that a function is a special relationship between an input and output (independent variable and dependent variable) in which each input is mapped consistently to an output.

Specific performance goals include:

- In the context of a vending machine simulation, students will determine if a mapping is a function.
- Students will describe what a function is in their own words.

The Vending Machine Task you engaged with was designed for students who have had previous experiences with function. When working with middle school students who have never heard of function, a slightly different strategy is needed.

Here is a copy of the Vending Machine Task designed for introducing the concept of function

Notice the first few pages identify which machine is and which is not a function, but students have to determine why those distinctions are being made.

Take a few minutes to engage with the middle school version of the task.

Here is the worksheet the students filled out and

Here is a description of the mappings that occur with each machine

Assignment

Watch the video Machines I and J Student Responses, of two groups (Hunter & Skyler and Leah & Rachel) engaging with the middle school student version of the applet.

While watching the video, focus on the students' language. Based on their responses, predict the students' responses for Machine K and N in the student version of the applet. Provide evidence from the video to justify your responses.

Machines I and J Student Responses - Group 1

Hunter & Skyler Explore Machines I and J

Q1. Attend to how Hunter and Skyler engaged with the applet to decide which machine was or was not a function.

An ideal answer should include technological actions that Hunter & Skyler use to explore Machines I & J, as well as their conclusion about each machine based on these actions. Focus on what they did and said. One example might be:

- For Machine I, Hunter and Skyler click on each color and take can several times. They pay attention to the fact that red gives two silver cans, blue gives green, green gives blue, and silver gives red consistently and come to the conclusion that Machine I is a function.
- For Machine J, Hunter and Skyler click on each can in succession at first, then one of them says, “you have to try them at least twice”. So they go back through and test each can at least twice to see that red produces red, blue produces blue and silver each time, silver produces silver, and green produces green. They talk about Machine I gives “the same product”, [i.e., two cans, same color], but Machine J gives a “different product”, [i.e., two cans of a different color] then conclude that Machine J is not a function because it is not the same “product”.

Q2. Interpret Hunter and Skyler's understanding of function based on their engagement with the applet. Use examples from the screencast as evidence to show how you know what they do or do not fully understand.

An ideal answer should include a coordination of how their engagement with the app led to their conclusion. One example might be:

- As Hunter and Skyler explored each machine they were careful to test each can on each machine multiple times and take can after. After they explored Machine I, they concluded it is a function because it is consistent, which shows they understand that a function should have a consistent output. After they explored Machine J, they focused on the difference between red on Machine I giving two silver cans and blue on Machine J giving a blue and silver can. They said that Machine I produced the “same product” when referring to the two silver cans produced, and said that the result of blue and silver in Machine J was a “different product”. This makes me think they are thinking of consistent in terms of the “product” must be of the same color, regardless of the number of cans. I noticed that the mapping actually says that for Machine J the result is blue and random, but Hunter and Skyler only got blue and silver each time they pressed blue in Machine J. I wonder if they would have seen at least one example of blue and another color, if that would help me understand their thinking better. I think they would have said not a function because it was not consistent or changed, rather than just the idea the cans were a different color. It might have pushed their thinking more because in my mind the fact that they got blue and silver each time was ‘consistent’, so why couldn’t both be a function, other than the fact the task said one was and one was not.

Q3. Create a scripted conversation between Hunter and Skyler that describes how you predict they will engage with Machines N and K to determine whether or not each is a function. For your script, denote engagement with the applet using parenthesis. See example below. Briefly justify your predicted script including evidence from the video.

Example:
Student 1: (clicks on the red cola button) It puts out green.
Student 2: Wow.
Student 1: (clicks on red cola button again) It still puts out green.
...

An ideal script should mimic the actions Hunter & Skyler took when engaging with Machine K & N. They were careful to select each can on each machine multiple times and take can each time. Their ideas focused on consistent outcomes that result in the same "product", where two cans of the same color were a function, but two cans of different colors, even when consistently the same (e.g., blue and silver each time), were not a function. Potential examples may include:

Table 8: This table can be used to write your scripted conversation for Hunter and Skyler. The first column lists the machines by letter. The second column is empty for your predictions. The third column is for your script with justification.

Machine	Prediction (Function/Non-Function)	Script with Justification Your script should include 1) precise language they will use when talking to each other; 2) exactly how the students will engage with the applet; and 3) provide insight into how their understanding of function will lead them to this conclusion.
K	Non-function	Note: Student on left (Hunter); Student on right (Skyler) Hunter: (clicking on red and taking can) Oh two red cans! (Assuming the first random pair is both red) That is like Machine I with the two silver cans, so it is a function. Skyler: Mate, I told you that you have to try each one at least twice. Hunter: (clicking on red and taking can at least twice in very quick succession; red & green first time; blue and silver next time; etc) These are not always the same. Wait, I'm confused. Skyler: Calm down, this one is doesn't give the same product each time, so it cannot be a function, but let's test the rest to be sure. Hunter: (Clicking on blue, silver, and green and taking can twice for each) Blue, blue, silver, silver, green, green. Skyler: So the rest of them give the can you press, but red still doesn't give the same product each time. It is not a function.
N	Non-function	Note: Student on left (Hunter); Student on right (Skyler)

		Hunter: (clicking on red, blue, silver and taking can two times in very quick succession) Red, red, blue, blue, silver, silver... Skyler: Ok, getting the same product each time. Hunter: (clicking on green and taking can at least twice in very quick succession) red and green, red and green. So this is like the blue and silver before. Skyler: Yeah, so this is not a function since it doesn't give us the same product each time.

Machines I and J Student Responses - Group 2

Leah & Rachel Explore Machines I and J

Q4. Attend to how Leah and Rachel engage with the applet to decide which machine was or was not a function.

An ideal answer should include technological actions that Leah and Rachel use to explore Machines I & J, as well as their conclusion about each machine based on these actions. Focus on what they did and said. One example might be:

- Machine I - The girls don't talk to each other much while they are clicking on each can, but they do test each can multiple times and take can each time. One of them says, "that is a lucky person" when she see that red gives two silver cans. After they test each can multiple times they conclude that Machine I is a function "because all of them give out different, consistent colors." Their work even says Machine I is a function because it gives out consistent color each time, even though they don't match the color selected.
- Machine J - Again the girls do not talk much. The same girl tests each can multiple times and takes can each time, similar to the first machine. She does act surprised when she presses blue and gets a blue and silver, then decides to test all the other cans multiple times before coming back to blue. She tests blue several times and gets two cans that vary in color each time. She then says to her partner, "I know the answer. It is not a function" because it produces two cans that are not consistent in color each time. Their work also supports this because they noted it does not give out consistent can color.

Q5. Interpret Leah and Rachel's understanding of function based on their engagement with the applet. Use examples from the screencast as evidence to show how you know what they do or do not fully understand.

An ideal answer should include a coordination of how their engagement with the app led to their conclusion. One example might be:

- Leah and Rachel are careful to test each can on each machine, followed by take can, each time. Their interaction with red on Machine I (two silver cans) and blue on Machine J (blue and random) help them focus on the idea that a function should produce a consistent outcome. They understand that a button

producing two cans is ok, provided the outcome is consistent each time. Their words when talking with the teacher include 'different colors', which could be like the first group that saw blue and silver each time to conclude not a function, but I don't think that is the case. Since the app gave them multiple different color pairs, I think they understood that it wasn't about the fact the two cans produced not being the same color, rather the important idea is that the two cans produced each time changed. I don't think the first pair of students grappled with that.

Q6. Create a scripted conversation between Leah and Rachel that describes how you predict they will engage with Machines K and N to determine whether or not each is a function. For your script, denote engagement with the applet using parenthesis. See example below. Briefly justify your predicted script including evidence from the video.

Example:
Student 1: (clicks on the red cola button) It puts out green.
Student 2: Wow.
Student 1: (clicks on red cola button again) It still puts out green.

An ideal script should mimic the actions Leah & Rachel took when engaging with Machine K & N. They were careful to select each can on each machine multiple times and take can each time. Their ideas focused on consistent outcomes, where two cans of the same color were a function, but two cans that change the combination of color each time were not a function. Potential examples may include:

Table 9: This table can be used to write your scripted conversation for Leah and Rachel. The first column lists the machines by letter. The second column is empty for your predictions. The third column is for your script with justification.

Machine	Prediction (Function/Non-Function)	Script with Justification Your script should include 1) precise language they will use when talking to each other; 2) exactly how the students will engage with the applet; and 3) provide insight into how their understanding of function will lead them to this conclusion.
K	Non-function	Note: Student on left (Leah); Student on right (Rachel) Leah: (clicks on red and takes can, two times; assume first pair is blue/blue, second pair is blue/red) Ok (after first pair). Wait, what?! (after second pair) Hey look at this. (clicks red a third time and gets red/silver) It is giving two cans, but they are not consistent in being the same color each time. Rachel: Then that is like Machine J, two cans but not consistent Leah: Yeah, it is not a function because it is not consistent.

N	Function	Note: Student on left (Leah); Student on right (Rachel) Leah: (clicks on red, blue, silver, and green then takes can, two times each) Ok, I know this is a function, look. (Repeats same process with Rachel looking) Rachel: Yeah, it is consistent, but the cans are not the same color like Machine I. Leah: Yeah, but when we talked with the teacher we said two cans that were consistent is a function. The color matching doesn't matter. Being consistent does. Rachel: (writing and talking) Ok, it is a function because it is consistent.

Q7. How has your own understanding of function been influenced by thinking about how middle school students develop the concept of function?

From the perspective of a teacher and ideal solution would include a focus on consistent/reliable/predictable outcome and deal with the idea of producing two cans/outcomes that are not consistent vs. that are consistent. Some examples may include:

- The vending machine applet pushed me to think about what it means to be a function in terms of what is selected/input and what results/output. When you get a consistent, single output from the same input then you are dealing with a function. The vending machine applet helps middle school students experience this in a familiar but non-math way.
- The set up of the vending machines each having one button that produces two cans presents a unique opportunity for middle school students to think about getting the same output each time you choose a given input. Each pair came to understand that two outputs are not allowed for it to be a function, which is good.
- I think the fact that the use of one machine that gives a single, consistent output each time sets students up to easily see that getting two cans/outputs each time is not a function. What I am wondering is about the machine that produces the same pair of cans each time. Machine I gives two silver cans consistently each time, so I wonder if middle school students would rely on the fact that it is a consistent outcome each time, so it would be a function. Or if they would say it is not a function just because it produces two cans. The students in this assignment went the path of two cans means not a function, but I could see a student arguing for the consistent output of two cans being a function.

5.3-I Commentary
Noticing and Predicting: Vending Machine Task (Introduction to the Concept of Function Version)

As you are preparing to begin a whole class discussion on the Vending Machine Task, teachers should have their worksheet with answers available and their laptops with the applet open to reference. In addition, we recommend passing back out the definitions of function teachers wrote prior to engaging with the task.

There are two parts to this discussion, the first is focused on discussing the specific machines in the applet; the second is focused on developing a definition of function. Furthermore, teachers will move in and out of small group and whole class discussion. Having your space set up for small groups that can stay arranged that way and still have a whole class discussion will be beneficial.

Introduction

In this assignment teachers will watch short video clips of pairs of HS students as they discuss Machines I and J from the Vending Machine Task. As a reminder, those machines behave as shown below.

Table 1: Machine I and J behaviors

Machine I	Machine J
Red Cola → 2 Silver	Red Cola → Red
Diet Blue → Green	Diet Blue → Blue & Random
Silver Mist → Red	Silver Mist → Silver
Green Dew → Blue	Green Dew → Green

We have included student names in all videos to make it easier for teachers to talk about the students' strategies. We have found that using names provides a sense of familiarity with the students and helps to mitigate the use of deficit language in discussing the students' work. Even so, it is important to remind teachers about honoring the power of the students' rough draft ideas as they discuss their current strategies and understandings.

Given that this assignment specifically asks teachers to attend to student strategies and interpret their understanding (the first two components noticing students' mathematical thinking), it might be helpful to remind them of the Noticing Students' Mathematical Thinking in Technology-Mediated Learning Environments Framework (see Module 0 for details).

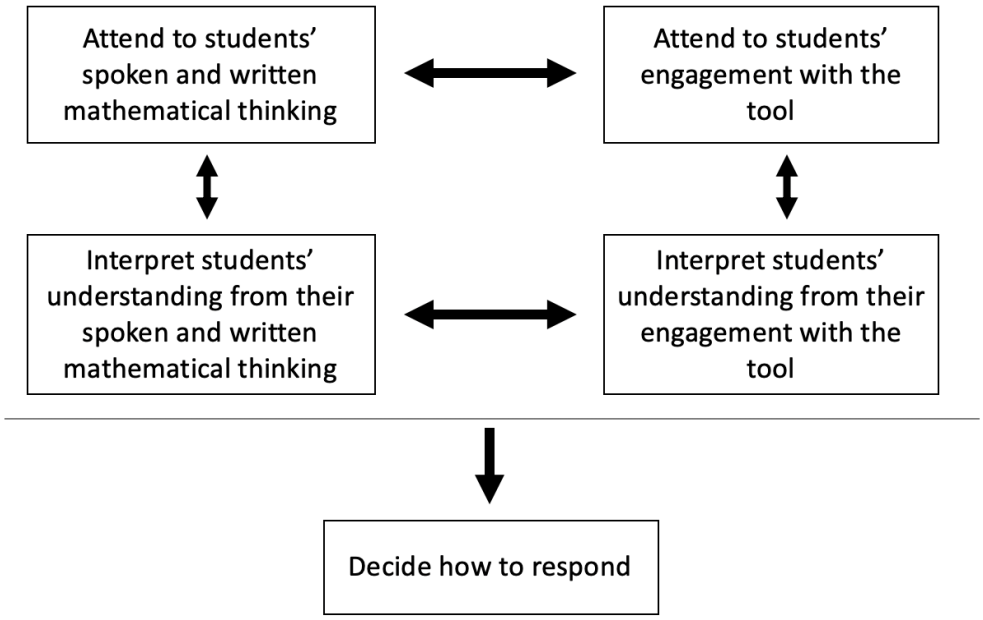

Figure 1. Noticing Students' Mathematical Thinking in Technology-Mediated Learning Environments Framework

Teacher Independent Work

Teachers should complete this assignment for HW, working independently, and then come prepared to share and discuss their responses the next class session. We recommend this being completed outside of class as teachers need time to watch the videos (possibly more than once) and reflect on them as they craft their responses to the prompts.

Whole Class Discussion

Begin the whole class discussion by displaying the framework for noticing students' mathematical thinking in a technology-mediated learning environment. Ask teachers what the different components - generally - might look like in the context of the Vending Machine task. In other words, what sorts of things should a teacher watch for / listen for when noticing student work with this task? Record these on the board as they are generated.

Responses might include:

- The way students test the individual buttons (i.e., clicking just once or more than once, using the take can button between clicks or not)
- Whether students are talking about the color of the cans, the number of cans, the consistency in the outputs when a particular input is pressed
- The personal definitions of function, input, or output they appear to be using as they test the cans
- Particular phrases they use to explain their thinking

Next, pick a few of the ideas listed (or combinations of them) and ask teachers what insight they provide to students' current (rough draft) understanding of function. For example, you might ask "What if you noticed a pair of students only testing the buttons on the machine once and recording the number of cans they see in the output. What interpretations might you make about their understanding of function?" (Avoid asking about the particular things they will be discussing related to the two groups of students they observed in the assignment.)

After this general discussion, move to talking specifically about the two groups of students they observed in the assignment—Group 1: Hunter and Skyler and Group 2: Leah and Rachel. It is helpful to record what teachers share for their attend (#1 and #4) responses, so everyone can look back at them when discussing their interpretations and predictions.

As teachers share their interpretations, make sure they provide justification from what students said or wrote AND their engagement with the machines. This coordination needs to be emphasized as an important feature of noticing (and monitoring) students' work on technology-enhanced mathematical tasks.

When you get to discussing teachers predictions, it is helpful to first poll whether or not they predicted the students would classify each machine as a function or not. This can be done informally or using a polling / survey technology. But a quick glance to know if there is agreement or not among their predictions will help you orchestrate this part of the discussion.

Possible discussion prompts include:

- (If there is not agreement) Using evidence from the video, explain your reasoning behind predicting students' X and X will decide machine X is (or is not) a function.
- You were asked to create a script for your prediction. How did you go about creating that script? What did you draw upon from the videos to do this?
- Why do you think I asked you to predict their responses to Machines E and I specifically?
- Are there any other machines you feel as if you can predict their responses to based on what you saw in the video? Why or why not?
- Are there specific ways in which the students interacted with each other that were important to your prediction? Why?
- In what ways might making predictions like this (both the mathematical and interactional aspects of the predictions) be helpful to you if you were monitoring a whole class of students working on this task?