



## The Function Concept - Functions and Non-Functions

**Total time:** 150 minutes but could be shortened to 75 minutes if needed. See page 2 for recommendations for a 75-minute module.

**Situating the Module within the PTMT Algebra Materials:** This module could be used alongside Chapter 6: Function of the PTMT Algebra Materials. Section 1: Experiencing Function is focused on leveraging one's experiences with common machines (i.e., vending machines) and prior mathematical experiences (i.e., transformations) to build an understanding of the function concept. Specifically, we use the Vending Machine task in Q19–23. If using both sets of materials, we recommend completing the PTMT Algebra Module Section 1 first. Using the materials provided here for 5.1 to accompany Q19–23 would be helpful, and the remaining tasks in this module follow naturally from where Section 1 ends.

**Overview of the Module:** This module draws on the PTMT Vending Machine GeoGebra applet designed to provoke a dilemma in relation to students' understanding of function. In this module teachers will first engage with the applet to make sense of function, domain, and range in a context that does not use traditional algebraic representations. Teachers will discuss affordances of the applet and discuss the definition of function. Teachers will then examine videos of pairs of high school students engaging with portions of the applet. Teachers will first analyze student responses to one particular machine as they focus on how the students' interaction with the machine led to their determination. For homework, teachers will watch more student pairs and will predict how the students would continue engaging with the applet on further machines—considering actions with the technology and students' verbal and written responses to task prompts. Finally, teachers will practice selecting and sequencing written student definitions. Additionally, there are optional homework assignments that introduces the idea of the concept images and challenge the teachers to consider how they might draw upon concept images in their anticipation of student thinking, and selecting and sequencing student strategies during whole class discussions.

**Important Note:** *This module has materials that allow for differentiation focused on whether middle/secondary school students are being introduced to the function concept for the first time or whether they are revisiting the function concept. Choose materials that align with the mathematical objectives of your course. For example, if your teachers will be introducing students to function for the first time, the introduction to the function concept materials might be helpful to include.*

### Module Goals:

- Consider how to use the Function Machine GeoGebra Applet, *Revisiting the Concept of Function*, to review the concept of function.
- Consider how to introduce the concept of function to students for the first time using the *Introduction to Function* applet
- Examine student practices when working in the Function Machine technological environment
- Consider how to facilitate the development of a mathematical definition (e.g., function) with a whole class



Table 1: Timeline of Tasks in the Module

<b>Timeline of tasks in the Module</b>	Day 0	5 min	5.1 Vending Machine Task (Revisiting the Concept of Function Version)
	Day 1	30 min	5.1 Discussion
		45 min	5.2 Noticing Student Thinking: Machine G of the Vending Machine Task
		Homework	5.3 Noticing and Predicting: The Vending Machine Task*
	Day 2	15 min	5.3 Discussion
		60 min	5.4 Selecting & Sequencing: Defining Function with the Vending Machine Task*
		Optional Homework	5.5 Anticipating Student Thinking: Concept Images of Function and the Vending Machine Task OR
		5.6 Orchestrating Whole Class Discussions: Defining Function with the Vending Machine Task*	

\* Indicates there are two versions of this task—one in which secondary school students are being introduced to the function concept for the first time and one in which secondary school students are revisiting the function concept.

#### MTE Preparation for Module 5:

- Engage with the Revisiting the Concept of Function applet.
- If you plan to use the 5.3 and/or 5.4 versions focused on introducing the function concept, it is important that you engage with the *Introduction to Function* applet.
- Read the commentaries and engage with the assignments you plan to use (including watching associated videos), and look over sample responses.
- Read pages 1–4 and 12–22 from Cooney, T. J., Beckmann, S., Lloyd, G. M., & Wilson, P. S. (2010). *Developing essential understanding of functions for teaching mathematics in grades 9–12*. National Council of Teachers of Mathematics.
- Read Dick, L., Lovett, J., McCulloch, A., Edgington, C., & Casey, S. (2020). Predicting students' mathematical thinking in a technology-mediated environment. *Journal of Technology and Teacher Education*, 28(3), 89–112.

#### Suggested Readings for Teachers:

- Lovett, J. N., McCulloch, A. W., Patterson, B. A., & Martin, P. S. (2020). Is this vending machine FUNCTIONing correctly? *Mathematics Teacher: Learning and Teaching PK-12*, 113(2), 132–139. <https://doi.org/10.5951/mtlt.2019.0087>
- Keiser, J. M. (2000). The role of definition. *Mathematics Teaching in the Middle School*, 5(8), 506–511.



Detailed Agenda for Module 5

Table 2: Agenda

	Description of Module Tasks	Facilitation Notes
<p style="text-align: center;"><b>Day 0</b></p>	<p><b>5.1 Vending Machine Task (Revisiting the Concept of Function Version) (5 minutes)</b></p> <p> Teacher devices</p> <p> <a href="#">Vending Machine – Revisiting the Concept of Function Version</a></p> <p>Before leaving class, teachers record their personal definition of function (without looking anything up) and are shown the applet that they will use for homework.</p>	<p>Keep the TEACHER written definitions, they will be used again at the beginning of the whole class discussion of the Vending Machine applet the following class.</p> <p>We recommend that you open the applet with the teachers and show them how to engage with it (i.e., be sure to click “take can”, you can equate this to removing the can from the vending machine, so it doesn’t get clogged, and you can see each output).</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>
	<p><b>5.1 Vending Machine Task (Revisiting the Concept of Function Version)</b></p> <p> Teacher devices</p> <p> <a href="#">Vending Machine – Revisiting the Concept of Function Version</a></p> <p> <a href="#">Worksheet</a></p> <p>For homework, teachers complete the electronic worksheet that uses the Vending Machine GeoGebra applet to build a deeper understanding of the function concept.</p>	<p>Provide a copy of the electronic worksheet to the teachers. Emphasize the importance of completing the task before doing the reading.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>



<b>Day 1</b>	<p><b>5.1 Discussion (30 minutes)</b></p> <ul style="list-style-type: none"> <li> Teacher devices Teacher Original Personal Definition</li> <li> <a href="#">Vending Machine – Revisiting the Concept of Function Version</a></li> <li> <a href="#">Vending Machine Task Mappings for Revisiting the Concept of Function Version</a></li> </ul> <p>Teachers discuss their experiences with the applet homework assignment in small groups, and after their discussions, teachers are given the opportunity to revise their personal definition of function that they recorded last class. Then engage in a whole class discussion to come to a class consensus on the definition of function.</p>	<p>Teachers should have their vending machine worksheet and laptops with the Vending Machine applet available during this discussion. When the teachers arrive, return their recorded personal definitions of function from the previous class. We suggest that teachers first discuss their worksheet responses in small groups and that afterward, using their original personal definitions as a reference, decide on a definition for function as a group, based on their experience with the Vending Machine applet. They should be prepared to discuss both their changes and reasons for making them.</p> <p>After discussing in small groups, the class comes together for a whole group discussion which includes both a discussion of the Vending Machine task and a final consensus for a class definition of function (see details in task commentary linked below).</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses. It will be also helpful for instructors to access Vending Machine Task Mappings for Revisiting the Concept of Function Version.</p>
	<p><b>5.2 Noticing Student Thinking: Machine G of the Vending Machine Task (45 minutes)</b></p> <ul style="list-style-type: none"> <li> Teacher devices</li> <li> <a href="#">Worksheet</a></li> </ul> <p>Teachers work in pairs or small groups to complete the assignment which involves watching videos of five pairs of students working through Machine G. The teachers are tasked with focusing on how the students' interactions with the applet lead them to their determination of function/non-function.</p>	<p>Provide a copy of the electronic worksheet to the teachers. We recommend facilitating a whole class discussion for Q6.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>



<b>Day 1 / Homework</b>	<p><b>5.3 Noticing and Predicting: The Vending Machine Task*</b></p> <p>*There are two versions of this task you can use to differentiate if you choose. The only difference is the student videos.</p> <p> Teacher devices</p> <p> <a href="#">Worksheet - Introduction to the Function Version</a></p> <p> <a href="#">Worksheet - Revisiting the Concept of Function Version</a></p> <p>Teachers individually watch two pairs of students engaging with Machines C &amp; D. For each pair, teachers attend and interpret the students' understanding of function and engagement with the applet. Teachers are then asked to predict how the students will respond to the following vending machines based on their analysis.</p>	<p>Provide a copy of the electronic worksheet to the teachers.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>
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<p><b>Day 2</b></p>	<p><b>5.3 Discussion</b> (15 minutes)</p> <p> Teacher devices</p> <p>Teachers participate in small group and whole class discussions about their noticing homework assignment.</p>	<p>We recommend facilitating a three-part discussion with teachers moving from the whole class, to small groups and back to the whole class. Begin with recording teachers' thoughts on student responses to each of the components of the NITE framework. Choose a few of the ideas listed and ask teachers what insight they provide to students' current (rough draft) understanding of function. After this general discussion, we suggest having teachers move to small groups to discuss their noticing of the two pairs of students they observed in the assignment. Next, we recommend polling teachers as to whether or not they predicted the students would classify each machine as a function or not and facilitating a whole class discussion.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>
	<p><b>5.4 Selecting &amp; Sequencing: Defining Function with the Vending Machine Task*</b> (60 minutes)</p> <p> Teacher devices</p> <p> <a href="#">Worksheet – Introduction to the Concept of Function Version**</a></p> <p> <a href="#">Vending Machine – Introduction to the Concept of Function Version</a></p> <p> <a href="#">Worksheet – Revisiting the Concept of Function Version**</a></p> <p> <a href="#">Vending Machine – Revisiting the Concept of Function Version</a></p> <p>Teachers work in pairs or small groups to practice selecting and sequencing high school students' rough draft definitions of function. They are given 20 definitions, asked to select 4-5 student definitions, sequencing them and discuss connections they would make during a whole class discussion.</p>	<p><b>**There are two versions of this task you can use to differentiate if you choose.</b> The only difference is the student definitions came from two different groups of students—those who had engaged with either the introduction to concept of function applet or the revisiting the concept of function applet.</p> <p>Teachers will need a physical copy of the worksheet. We recommend either cutting out, or providing scissors so they can, the definitions on the last page so they are easily sorted.</p> <p>After the teachers finish, we suggest having each pair/group display their sequencing, share their justifications and the connections they plan to make. Facilitate this discussion amongst the teachers. Emphasize that there is not a “correct” selection or sequencing; it is just important that the teacher is intentional.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>



<p>Day 2 / Homework</p>	<p><b>5.5 Anticipating Student Thinking: Concept Images of Function and the Vending Machine Task</b></p> <p> Teacher devices</p> <p> <a href="#">Worksheet</a></p> <p>Teachers work in pairs or a small group to complete a Desmos assignment that asks them to consider how a pair of students' concept images of function led to their determinations and challenges the teachers to consider how they might draw upon concept images in their anticipation of student thinking, and selecting and sequencing student strategies during whole class discussions.</p> <p>-----OR-----</p> <p><b>5.6 Orchestrating Whole Class Discussions: Defining Function with the Vending Machine Task</b></p> <p> <a href="#">Worksheet - Introduction to the Concept of Function Version**</a></p> <p> <a href="#">Worksheet - Revisiting the Concept of Function Version**</a></p> <p>This assignment builds upon 5.4 Selecting &amp; Sequencing Definitions of Function and asks teachers to script out an ideal orchestration of a whole class discussion towards a shared definition of function.</p>	<p>Provide a copy of the electronic worksheet to the teachers.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p> <p><b>**There are two versions of this task you can use to differentiate if you choose.</b></p> <p>Provide a copy of the electronic worksheet to the teachers.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>
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