Module 4 Overview Document

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	Homework	4.1 Avi & Benita's Repair Shop
		Homework	4.2 Noticing Student Thinking: Avi & Benita's Repair Shop
	Day 1	10 min	4.1 Discussion
		20 min	4.2 Discussion
		45 min	4.3 Selecting & Sequencing: Avi & Benita's Repair Shop
	Day 2	75 min	4.4 Three Animals Race
	Day 3	45 min	4.5 Noticing Student Thinking: Three Animals Race
		30 min	4.6 Assessing and Advancing Student Thinking: Three Animals Race

4.6 Facilitation Notes

As with 4.5, the student work included in this task comes from high school students in an Integrated Math 2 course. The students engaged in a version of the Three Animals Race Task using a slightly different activity design. Rather than having separate GeoGebra files linked throughout the task, the students' version collected the files into a GeoGebra book. The Three Animals Race GeoGebra Book has four pages that include the simulation (Page 1), the spreadsheet (Page 2), a STOP page (Page 3), and a page that includes a correct, completed spreadsheet and graph view (Page 4).

Three Animals Race GeoGebra Book

The purpose of this assignment is to provide teachers an opportunity to notice students' written work which shows different interpretations of various changing speeds over time (i.e., describing rates of change). After attending to and interpreting student responses, teachers decide how to respond by determining purposeful assessing and advancing questions that they would ask three different pairs of students.

We recommend having teachers complete this task as a jigsaw (i.e., small groups attend, interpret and decide one pair of students' mathematical thinking, then groups switch and share their noticing with each other). Encourage teachers to keep the task open in case they want to model what the students were doing to obtain their output. When comparing their noticing of the three student pairs, it is important to ensure teachers are discussing how students are interpreting starting distances, common differences and/or ratios and how they are building their tables.

Note: If you are only using 4.6, you may choose to stop the teachers after their anticipations in Q1 and show one or both of the student video clips from 4.5. You could then have the teachers discuss how their anticipations were similar or different from the students' approaches and give the teachers an opportunity to refine their anticipations before moving on to the jigsaw component.

Sample Responses (pp. 2–7) are below.

4.6 Sample Responses
Assessing and Advancing Student Thinking: Three Animals Race

The purpose of the Three Animals Race Task is to help students distinguish between, and be able to describe, the different ways distance would change over time (i.e., the rate of change) for linear, quadratic, and exponential relationships.

Three Animals Race Task

The learning and performance goals aligned with this task include:

- Students will understand that not all contexts can be described as changing linearly or exponentially.
- Students will explain the difference between the three animals in the race using their rates of change. Specifically, the turtle is moving away from the starting line at a constant rate (linear), rabbit is moving away from the starting line at a constantly additive rate (quadratic), and alligator is moving away from the starting line at a constantly multiplicative rate (exponential).
- Students will model the race and determine a winner by building a table in a spreadsheet using what they know about the animals' rates of change and distance from the starting line.

Q1. How do you anticipate high school students will use the simulation and spreadsheet to determine the winner of the race? Who will they pick as the winner? Why?

- I anticipate students will use the simulation to see how fast the animals are moving throughout the race. They will drag the slider until they see which animal crosses the finish line first. They will use the spreadsheet to estimate the time it takes for each animal to get to the finish line. I think they will pick the alligator because he moves faster throughout the race and crosses the finish line first in the simulation.
- I think they will drag the slider to run the simulation multiple times before slowing down and looking at the last few seconds before the animals cross the finish line. I think they will choose rabbit because he is just a 'hare' (get it 🙂) ahead of turtle at 13 seconds, with alligator behind both of them.

Imagine a pair of students, D'Niya and Lukas, have just announced to you that they know who won the race and asked you to come over and see their work. The spreadsheet they created is shown below (Figure 1). Examine it carefully.

	A	B	C	D
1	time (sec)	Turtle Distance from Start	Rabbit Distance from Start	Alligator Distance from Start
2	1	50	1	0
3	2	53	2	0
4	3	56	3	0
5	4	59	4	0
6	5	62	5	0
7	6	65	6	1.5
8	7	68	7	4.5
9	8	71	8	6
10	9	74	9	7.5
11	10	77	10	9
12	11	80	11	10.5
13	12	83	12	12
14	13	86	13	13.5
15	14	89	14	15
16	15	92	15	16.5
17	16	95	16	18
18	17	98	17	19.5
19	18	101	18	
20			19	
21			20	
22			21	
23			22	

Figure 1: D'Niya and Lukas' Spreadsheet

Q2. Attend to D'Niya and Lukas' thinking about the three animals in the race based on what they recorded in their spreadsheet.

- They have the turtle winning the race with this spreadsheet. They have the alligator going up 1.5 each second, the rabbit going up one each second, and the turtle going up 3 each second. They also have the turtle starting at 50 in 1 second instead of 0 seconds.
- Based off of their work, they see the rabbit is moving at a constant rate, making it a linear graph. The turtle started at the 50 meter line and is moving 3 meters per second, still at a constant speed but faster than the rabbit at 1 meter per second.

They have the alligator moving at an exponential rate of change, since he does not move until 6 seconds and then gradually builds up speed.

Q3. Based on what you attended to in their spreadsheet, Interpret D'Niya and Lukas's current understanding of the ways that each animal's distance from the starting line is changing.

- They have the alligator going up 1.5 each second, the rabbit going up one each second, and the turtle going up 3 each second. Their table values show they understand how fast each animal is moving as they run the race, but they are thinking this is also the total distance they have traveled each second. This might be because for turtle going at a constant rate it is easy for them to just add three each time and get the right answer for how far he is from the starting line. I think their understanding of how the speed plays into how far they are from the finish line is still in development. They have not yet figured out they have to add the distance traveled (based on the rate of change) between each second to get the distance for rabbit and alligator.
- Their work shows that they think that Rabbit and Turtle are moving at a constant speed away from the starting line while the Alligator is moving faster and faster as it goes away from the starting line based off of their table.

Q4. What assessing questions would you like to ask D'Niya and Lukas? Why?

- What calculations did you use to get these distances? Because I want them to explain how they got the values in the table to understand more about why they think the speed of each animal is the same as the distance from the starting line. Plus, I hope it would have them think more about how well the values in the table fit each scenario of how the animals are moving related to distance. Who do you think will win? Because this will get them to explain how they are using the values in the table to figure out the winner.
- "Why is the rabbit moving slower than the turtle?" Because based off their table they said the rabbit is slower than the turtle, so I want to make them think about why their calculations got that conclusion to hopefully see that rabbit is not moving at a constant speed.

Q5. What advancing questions would you like to pose to D'Niya and Lukas? Explain what you hope your question(s) would achieve.

Note: Consider how you might leverage the technology (i.e., the simulation or the tools in the spreadsheet) in your question(s).

- Is there any other way to look at the spreadsheet or the simulation to figure out which animal would win the race? Because I want them to compare the two

representations to have them think about if what they see in the simulation matches what they have in their table for each animal.
- "When looking at the simulation, does your table represent the movement of the rabbit?" I hope this would let students see that their calculation of the rabbit is wrong and they need to see visually why that is.

Another pair of students, Essence and Jovani, have now announced to you that they know who won the race and asked you to come over and see their work. The spreadsheet they created is shown below (Figure 2). Examine it carefully.

	A	B	C	D
1	time (sec)	Turtle Distance from Start	Rabbit Distance from Start	Alligator Distance from Start
2	1	53	1	0
3	2	56	3	0
4	3	59	6	0
5	4	62	10	0
6	5	65	15	0
7	6	68	21	1.5
8	7	71	28	3.75
9	8	74	36	7.13
10	9	77	45	12.19
11	10	80	55	19.78
12	11	83	66	31.17
13	12	86	78	48.24

Figure 2: Essence and Jovani's Spreadsheet

Q6. Attend to Essence and Jovani's thinking about the three animals in the race.

- Their table values show that they understand the correct starting distance for turtle at time 0 is 50, and for rabbit and alligator it is 0 at 0 seconds. They also have accurate values for how far each animal is from the starting line per second, which means they have thought about how fast each animal is moving per second to find how far they went in during that time to get the total distance traveled.
- The values in their table show that turtle moves at a constant rate of 3 meters each second. The rabbit moves by 2 meters then 3 then 4 and so on, which was added to the previous distance to get the total distance at each time. The alligator does not move for a while then goes faster and faster because his speed is multiplied by 1.5 each second to get the distance traveled, then added to the previous distance to get the total distance at each time.

Q7. Interpret Essence and Jovani's understanding of the ways that each of the animals' distance from the starting line is changing.

- They understand the race starts at 0 seconds and not 1 second. They understand that turtle moves 3 meters each second, so they need to add 3 to each previous distance to get the new distance. They understand that rabbit is increasing speed by 1 meter for each second (like 1 meter per second, 2 meters per second, 3 meters per second, etc.), so they knew to increase his distance between intervals by his speed for that interval to get total distance traveled. For alligator they understand he does not move for the first five seconds, and his speed starts at 1.5 meters per second to start, but it gets multiplied by 1.5 each second. They used this to calculate his speed during each interval and find the total distance traveled.
- The turtle moves constant for a linear graph. The rabbit moves faster every second but in a constant way (+1) like a quadratic graph. The Alligator doesn't move at first but then moves really fast, in a exponential way.

Q8. What assessing questions would you like to ask Essence and Jovani? Why?

- What process did you use to determine the distance from the starting line for each animal? How did their rate of change help you? Because I want them to explain the reasoning behind how they got their table values.
- "Are all the animals moving the same speed? How do you know?" this will get them to reason why they are not moving at the same speed and connect to each type of rate of change.

Q9. What advancing questions would you like to pose to Essence and Jovani? Explain what you hope your question(s) would achieve.

Note: Consider how you might leverage the technology (i.e., the simulation or the tools in the spreadsheet) in your question(s).

- Who do you think will win the race based on your table? How does this compare to the simulation? Because I want them to think about what they are able to 'see' from each representation. Maybe even talk about which one they like better to help figure out the winner.
- "What graph would each animal represent and why?" to help students see what each type of graph looks like in a table that is built from using different types of rate of change and match the animal to that graph.

Another pair of students, Avery and Michael, have now announced to you that they know who won the race and asked you to come over and see their work. The spreadsheet they created is shown below (Figure 3). Examine it carefully.

	A	B	C	D
1	time (sec)	Turtle Distance from Start	Rabbit Distance from Start	Alligator Distance from Start
2	1	50	1	0
3	2	53	3	0
4	3	56	6	0
5	4	59	10	0
6	5	62	15	0
7	6	65	21	1.5
8	7	68	28	4.5
9	8	71	36	9
10	9	74	45	15
11	10	77	55	22.5
12	11	80	66	31.5
13	12	83	78	42
14	13	86	91	54
15	14	89	105	67.5
16	15	92		82.5
17	16	95		99
18	17	98		117
19	18	101		

Figure 3: Avery and Michael's Spreadsheet

Q10. Attend to Avery and Michael's thinking about the three animals in the race.

- They started turtle at 50 meters at 1 second instead of 0, but they started rabbit an alligator at the correct position for time zero since their distances for each at 1 second are correct. They have the turtle moving up at a constant rate of 3. Rabbit is moving by adding one to the previous movement constantly. The alligator is moving up but it took me a while to figure out how they were coming up with their numbers. I finally figured out that they were getting alligators speed for each second by multiplying 1.5 times 2, then 3, then 4, etc and adding it to the previous distance to get the total distance traveled. Based on their table rabbit will win the race.

- Based off their table they state that the rabbit will win the race. The turtle is moving at a constant pace. The rabbit is moving faster than everyone and going up by multiples. The alligator started late and moved faster as time went on but did not win.

Q11. Interpret Avery and Michael's understanding of the ways that each of the animals' distance from the starting line is changing.

- They understand that turtle is moving at a constant rate of 3 meters per second away from the starting line because they added 3 meters to each previous distance. They also understand that rabbit's distance from the starting line is increasing by 1 meter for every additional second, so he is speeding up at a constant rate which gets him further away from the starting line more and more quickly. For alligator they understand his rate starts at 1.5 meters per second and increases, but they are adding multiples of 1.5 to the previous distance, instead of multiplying by 1.5 each time to get the rate during each interval to determine the total distance. Overall they get that they have to add the distance traveled in each second, based on the rate for each animal, but they misinterpreted alligator.
- Their understanding is that the turtle will move at a constant rate. The rabbit will move faster as time goes on, constantly. The alligator will start late but increase speed very fast.

Q12. What assessing questions would you like to ask Avery and Michael? Why?

- How do you know turtle is at 50 meters at 1 second? Because I am hoping they will realize their error during their explanation. What process did you use to calculate the distance for each animal? Because I want them to explain their thinking in detail for each animal, especially alligator. Which animal do you think would win based on your table? Because I want them to explain how they are using the table to determine the winner.
- "Why do you think the rabbit would win if the alligator starts moving really fast?" This will help students to look at their table and the simulation and see if they match up.

Q13. What advancing questions would you like to pose to Avery and Michael? Explain what you hope your question(s) would achieve.

Note: Consider how you might leverage the technology (i.e., the simulation or the tools in the spreadsheet) in your question(s).

- Look back at the simulation compared to your table. Based on the spreadsheet and simulation who do you think will win? Because I want them to think about

what they are seeing in the simulation about how each animal is moving versus their table values. I am hoping this will get them to think more about alligator in the table.

- "If your simulation shows the alligator winning, but your table shows the rabbit winning, what conclusion can be made about the calculations? Which one is correct, the simulation or the table? Why?" They need to see that their calculation could be off somewhere making the rabbit win.