



Module 4 Overview Document

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	Homework	4.1 Avi & Benita's Repair Shop
		Homework	4.2 Noticing Student Thinking: Avi & Benita's Repair Shop Task
	Day 1	10 min	4.1 Discussion
		20 min	4.2 Discussion
		45 min	4.3 Selecting & Sequencing: Avi & Benita's Repair Shop
	Day 2	75 min	4.4 Three Animals Race
	Day 3	45 min	4.5 Noticing Student Thinking: Three Animals Race
		30 min	4.6 Assessing and Advancing Student Thinking: Three Animals Race

4.1 Facilitation Notes

The task used in 4.1–4.3 is a Desmos task called Avi & Benita's Repair Shop. Create a class code for the Desmos task and provide the link to your teachers. We recommend asking teachers to log in so that the task will appear in their history and they can revisit it at any time. In addition, they must be logged in to receive any feedback you provide.



Avi & Benita's Repair Shop

The timeline suggests that teachers engage in the task as learners for homework. When the teachers return, we recommend showing the Desmos Teacher Dashboard with the teachers' responses and then utilizing the snapshot and presentation tools to model use of the Teacher Dashboard while facilitating a brief discussion of the task. As time allows, we recommend discussing how the different representations used in the task (i.e., tables and graphs) supported their own sensemaking about the similarities and differences between the two situations in this particular task.



4.1 Sample Responses

Avi & Benita's Repair Shop

Below we share some sample responses to key pages of the Avi & Benita's Repair Shop Task.

Page 2:

Complete the table.

Day	Avi's Rule (Dollars)	Benita's Rule (Dollars)
1	0.01	100
2	0.02	200
3	0.04	300
4		

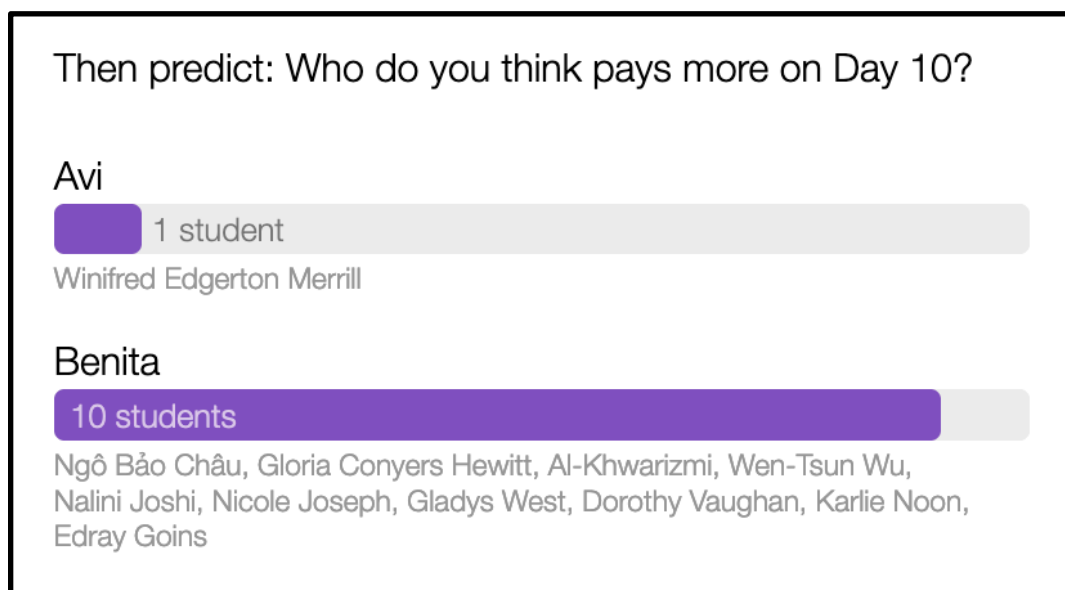
The table shows how much you'll earn with each rule.

Complete the table for Day 4.

Then predict: Who do you think pays more on Day 10?

Page 2 of the Avi & Benita's Repair Shop Task

This page will generate a poll as your teachers respond.



Sample Poll Results from Page 2

Module 4: Comparing and Contrasting Linear, Quadratic, and Exponential Rate of Change
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Once teachers answer the initial question by selecting Avi or Benita, they will be prompted to provide an explanation.

Complete the table.

Day	Avi's Rule (Dollars)	Benita's Rule (Dollars)
1	0.01	100
2	0.02	200
3	0.04	300
4		

The table shows how much you'll earn with each rule.
Complete the table for Day 4.
Then predict: Who do you think pays more on Day 10?

Avi Benita

Explain your thinking.

Share With Class

Additional Prompts on Page 2

Below is sample reasoning from a teacher who selected Avi.

Complete the table.

Day	Avi's Rule (Dollars)	Benita's Rule (Dollars)
1	0.01	100
2	0.02	200
3	0.04	300
4	0.08	400

The table shows how much you'll earn with each rule.
Complete the table for Day 4.
Then predict: Who do you think pays more on Day 10?

Avi Benita

Explain your thinking.

avi is exponential and benita is linear so I know that doubling something will b larger than something going up by 100 each time the other doubles

Edit my response

Reasoning Sample Teacher Response for Selecting Avi on Page 2



Below are several reasoning samples from teachers who selected Benita.

Complete the table.

Day	Avi's Rule (Dollars)	Benita's Rule (Dollars)
1	0.01	100
2	0.02	200
3	0.04	300
4	0.08	400

The table shows how much you'll earn with each rule.

Complete the table for Day 4.

Then predict: Who do you think pays more on Day 10?

Avi Benita

Explain your thinking.

Benita's is plus \$100 every day while Avi's is doubling every day. Although long-term Avi's would give you more across 10 days Benita's would be more.

Edit my response

Complete the table.

Day	Avi's Rule (Dollars)	Benita's Rule (Dollars)
1	0.01	100
2	0.02	200
3	0.04	300
4	0.08	400

The table shows how much you'll earn with each rule.

Complete the table for Day 4.

Then predict: Who do you think pays more on Day 10?

Avi Benita

Explain your thinking.

I believe that Benita pays more on day 10 because she is adding \$100 to you pay per day. This means on day 10, you would get paid \$1000. Avi's rule is that she multiplies the previous day's pay by 2 and that is what you get paid. When you do this, on day 10, you would get paid \$5.12.

Edit my response

Complete the table.

Day	Avi's Rule (Dollars)	Benita's Rule (Dollars)
1	0.01	100
2	0.02	200
3	0.04	300
4	0.08	400

The table shows how much you'll earn with each rule.

Complete the table for Day 4.

Then predict: Who do you think pays more on Day 10?

Avi Benita

Explain your thinking.

Avi's rule is doubling each day while Benti increases by 100 everyday. While Avi's rule increases a little amount in the beginning it will soon start to grow to very large amounts because its exponentially growing. On day 10, Benti will pay 1,000 dollars while Avi will only pay 5 dollars.

Edit my response

Three Different Reasoning Sample Teacher Responses for Selecting Benita on



Page 3:

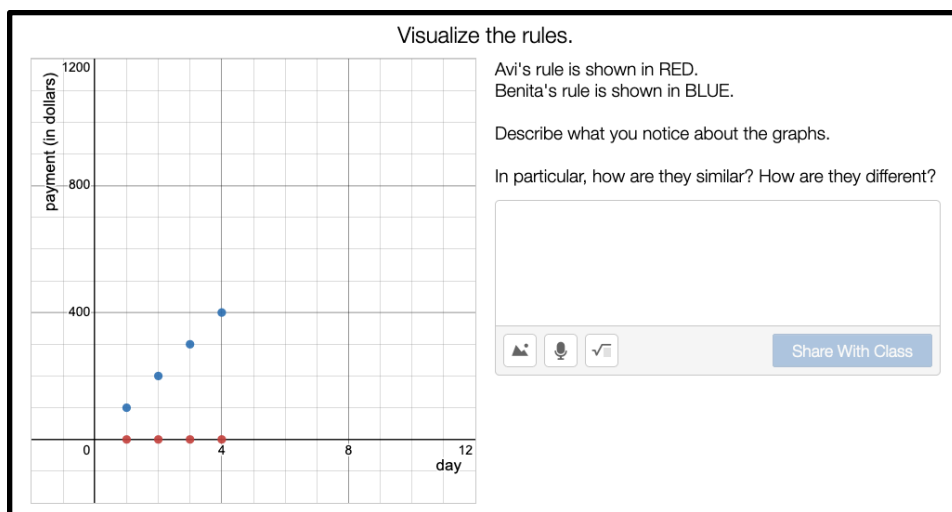
Page 3 asks teachers to extend the table. *Note: Gray boxes provided in prompt, white boxes are a sample student response*

Day	Avi's Rule (Dollars)	Benita's Rule (Dollars)
1	0.01	100
2	0.02	200
3	0.04	300
4	0.08	400
5	0.16	500
6	0.32	600
7	0.64	700
8	1.28	800
9	2.56	900
10	5.12	1000

Sample Teacher Response for Page 3

Page 4:

The graph on page 4 will plot all the points from page 3. So each teacher will have the same initial values (gray boxes from the figure above) and then graph the points the teachers calculate (white boxes from the figure above).



Page 4 if teachers did not complete Page 3



Below are sample teacher responses from Page 4.

Visualize the rules.

day	payment (in dollars)
0	0
1	100
2	200
3	300
4	400
5	500
6	600
7	700
8	800
9	900

day	payment (in dollars)
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0

Avi's rule is shown in RED.
Benita's rule is shown in BLUE.

Describe what you notice about the graphs.

In particular, how are they similar? How are they different?

I notice that Benitas graph is increasing at a constant rate. While it appears that Avi graph is not increasing but rather a straight line across the x-axis. While Benita has a slope of 100 and Avi has a slope of 0, both lines look to be linear.

[Edit my response](#)

Three other students' responses would show up here.

Visualize the rules.

day	payment (in dollars)
0	0
1	100
2	200
3	300
4	400
5	500
6	600
7	700
8	800
9	900

day	payment (in dollars)
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0

Avi's rule is shown in RED.
Benita's rule is shown in BLUE.

Describe what you notice about the graphs.

In particular, how are they similar? How are they different?

Benita's is a linear graph so it's increasing at the same slope each time. Avi's at some point will be an exponential graph because at some x it will increase very quickly. They similar cause they're both increasing but different because it's two different fucntions.

[Edit my response](#)

Three other students' responses would show up here.

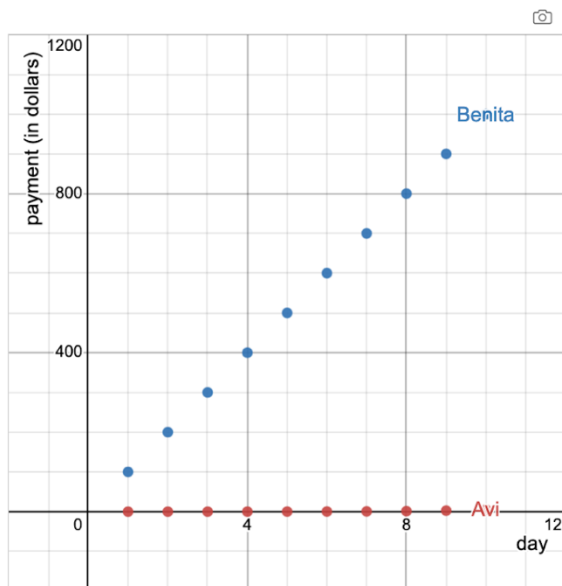
Sample Teacher Responses from Page 4

Module 4: Comparing and Contrasting Linear, Quadratic, and Exponential Rate of Change

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Visualize the rules.



Avi's rule is shown in RED.

Benita's rule is shown in BLUE.

Describe what you notice about the graphs.

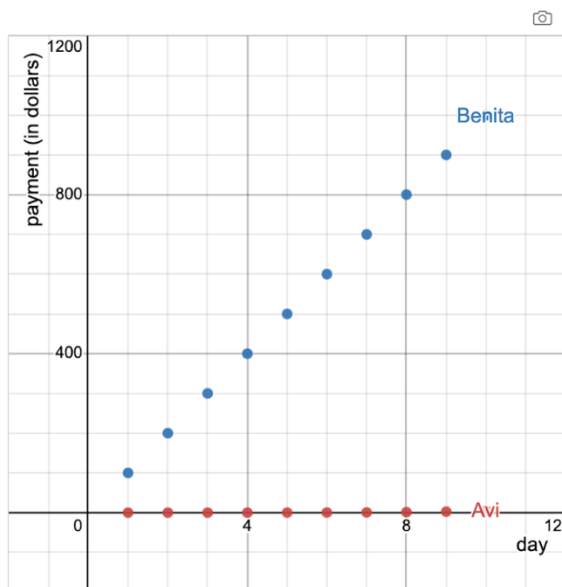
In particular, how are they similar? How are they different?

Similarly, they are both straight lines. Both of the graphs are "technically" increasing, even if Avi's is only by cents right now. Benita's graph on the other hand is a constant linear equation. Because the payment is in hundreds of dollars, Benita's graph is showing a constant increase while Avi's isn't off the ground, or x-axis.

Edit my response

Three other students' responses would show up here.

Visualize the rules.



Avi's rule is shown in RED.

Benita's rule is shown in BLUE.

Describe what you notice about the graphs.

In particular, how are they similar? How are they different?

I notice that Benita's graph is increasing at a constant rate. While it appears that Avi's graph is not increasing but rather a straight line across the x-axis. While Benita has a slope of 100 and Avi has a slope of 0, both lines look to be linear.

Edit my response

Three other students' responses would show up here.

Sample Teacher Responses from Page 4



Page 5:

Page 5 asks teachers to sketch the graph extending to day 20. Below are sample teacher responses.

The graph displays two functions: Benita's Rule (linear) and Avi's Rule (exponential). The x-axis represents 'day' from 0 to 20, and the y-axis represents 'payment (in dollars)' from 0 to 6000. Benita's Rule starts at (0,0) and increases linearly to approximately (20, 2000). Avi's Rule starts at (0,0) and increases exponentially, reaching approximately (20, 5242.88).

Which rule do you think will pay more on Day 20?

Use math to support your answer.

Use the sketch tool on the graph if that helps to illustrate your thinking.

Avi's Rule
12 students
Ngô Bảo Châu, Gloria Conyers Hewitt, Al-Khwarizmi, Wen-Tsun Wu, Nalini Joshi, Nicole Joseph, Gladys West, Dorothy Vaughan, Karlie Noon, Zhang Heng, Edray Goins, Winifred Edgerton Merrill

Benita's Rule
0 students

Day 20

The graph displays two functions: Benita's Rule (linear) and Avi's Rule (exponential). The x-axis represents 'day' from 0 to 20, and the y-axis represents 'payment (in dollars)' from 0 to 6000. Benita's Rule starts at (0,0) and increases linearly to approximately (20, 2000). Avi's Rule starts at (0,0) and increases exponentially, reaching approximately (20, 5242.88).

Which rule do you think will pay more on Day 20?

Use math to support your answer.

Use the sketch tool on the graph if that helps to illustrate your thinking.

Avi's Rule **Benita's Rule**

Explain your thinking.

I believe Avi's Rule will pay more on Day 20 because Benita's rule will pay \$2000 on day 20 since it is a linear function. However, since Avi's rule is an exponential function, it will pay \$5,242.88 on day 20

[Edit my response](#)

Three other students' responses would show up here.

Sample Teacher Responses for Page 5



Day 20

Which rule do you think will pay more on Day 20?

Use math to support your answer.

Use the sketch tool on the graph if that helps to illustrate your thinking.

Avi's Rule Benita's Rule

Explain your thinking.

With Avi's rule, day 15 is where you see the change in the graph. Because you are still multiplying the last day by 2, the graph is increasing at an exponential rate. With a graph growing exponentially, it will always be greater than a linear one given an infinite amount of time.

Edit my response

Three other students' responses would show up here.

Sample Teacher Response for Page 5

Page 6:

Page 6 asks teachers to complete the table that extends to day 20. Below are sample teacher responses. *Note: top two gray boxes are provided by piping the teacher responses from page 3, white boxes are a sample student response from this page*

Responses Overlay

Show Original

Complete the table, then press "Try It."

Day	Avi's Rule (Dollars)	Benita's Rule (Dollars)
9	2.56	900
10	5.12	1000
11	10.24	1100
12	20.48	1200
13	40.96	1300
...
20	5242.88	2000

Sample Teacher Response for Page 6



Page 7:

Page 7 asks teachers to compare the graphs. Below are sample teacher responses.

Describe the graphs.

Day	Payment (in dollars)
0	0
5	500
10	1000
15	1500
20	2000

Day	Payment (in dollars)
0	1
5	1.05
10	1.10
15	1.28
20	5300

Avi's rule is shown in RED. Benita's rule is shown in BLUE.

Now what do you notice about the graphs?

In particular, is there anything you're surprised by?

Benita's graph increases at a steady rate while Avi's graph starts out slow then starts to increase extremely fast. What surprised me is that Avi's graph my start at just a penny a day but by day 20 you are making more than double what you would with Benita.

Edit my response

Three other students' responses would show up here.

Describe the graphs.

Day	Payment (in dollars)
0	0
5	500
10	1000
15	1500
20	2000

Day	Payment (in dollars)
0	1
5	1.05
10	1.10
15	1.28
20	5300

Avi's rule is shown in RED. Benita's rule is shown in BLUE.

Now what do you notice about the graphs?

In particular, is there anything you're surprised by?

I notice that Avi's line is no longer linear, while Benita's rule is still increasing by a constant rate. Avi's graph is beginning to look more like a expontial graph as time continues. I was suprised to see that by day 19 Avi's rule begins to be better than Benita's. I was not suprised to see that Avi's graph began to increase tremendously faster.

Edit my response

Three other students' responses would show up here.

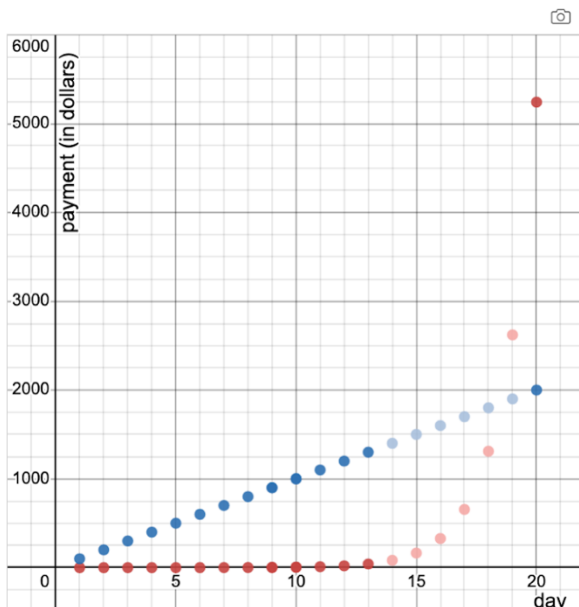
Sample Teacher Responses for Page 7

Module 4: Comparing and Contrasting Linear, Quadratic, and Exponential Rate of Change

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Describe the graphs.



Avi's rule is shown in RED. Benita's rule is shown in BLUE.

Now what do you notice about the graphs?

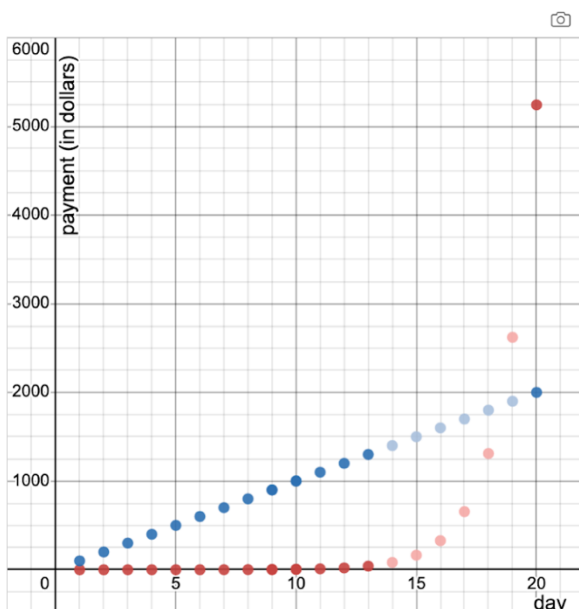
In particular, is there anything you're surprised by?

I notice that my theory that Avi's graph was an exponential graph and that Benita's rule was a linear graph was correct. I notice that her rule also starts super small but grows quickly. I was surprised that Avi's rule did not grow faster than it did. It was only by day 19 and 20 that Avi's rule was more than Benita's rule.

Edit my response

Three other students' responses would show up here.

Describe the graphs.



Avi's rule is shown in RED. Benita's rule is shown in BLUE.

Now what do you notice about the graphs?

In particular, is there anything you're surprised by?

Avi's rule now is now visibly exponential, so it's clear that Benita's rule is less expensive. I am surprised by how quickly the graph multiplied.

Edit my response

Three other students' responses would show up here.

Sample Teacher Responses for Page 7



Page 8:

Page 8 asks teachers to consider when each rule is preferred. Below are sample teacher responses.

A New Hire

Avi and Benita's newest employee, Cal, asks you for advice:

"For what days should I get paid by Avi's rule? And for what days should I get paid by Benita's rule?"

What advice would you give to Cal?

Based on the graphs I would get paid by Benita's rule from day 1-18 and then Avi's rule from 18-up

[Edit my response](#)

Three other students' responses would show up here.

A New Hire

Avi and Benita's newest employee, Cal, asks you for advice:

"For what days should I get paid by Avi's rule? And for what days should I get paid by Benita's rule?"

What advice would you give to Cal?

I would tell Cal that for the first 18 days, he should get paid by Benita's rule. This is because he would get paid more this way. He would get paid \$100 the first day, \$200 the 2nd, \$1800 the 18th. However, starting on day 19, he should get paid with Avi's rule. This is because he will be getting paid a ton more. He will make close to three thousand more dollars with Avi's rule on day 20. She will make close to \$8000 more on day 21.

[Edit my response](#)

Three other students' responses would show up here.

Sample Teacher Responses for Page 8



Page 9:

Page 9 asks teachers to consider which rule they would pick if they could only choose one rule. Below are sample teacher responses for Page 9.

Bonus Challenge

Avi and Benita just hired another employee, Dee.

Instead of letting her decide each day which rule she'd like to get paid by, they're making Dee pick one rule from the beginning (and she has to stick with it).

What advice would you give to Dee? Is there anything you need to know before you can give sound and helpful advice?

If you are planning on staying with the company long-term choose Avi's rule but if you are planning on staying less than 18 days choose Benita's rule. Benita's rule pays more up until day 19 so after day 19 you would get paid way more with Avi's rule

Edit my response

Three other students' responses would show up here.

Bonus Challenge

Avi and Benita just hired another employee, Dee.

Instead of letting her decide each day which rule she'd like to get paid by, they're making Dee pick one rule from the beginning (and she has to stick with it).

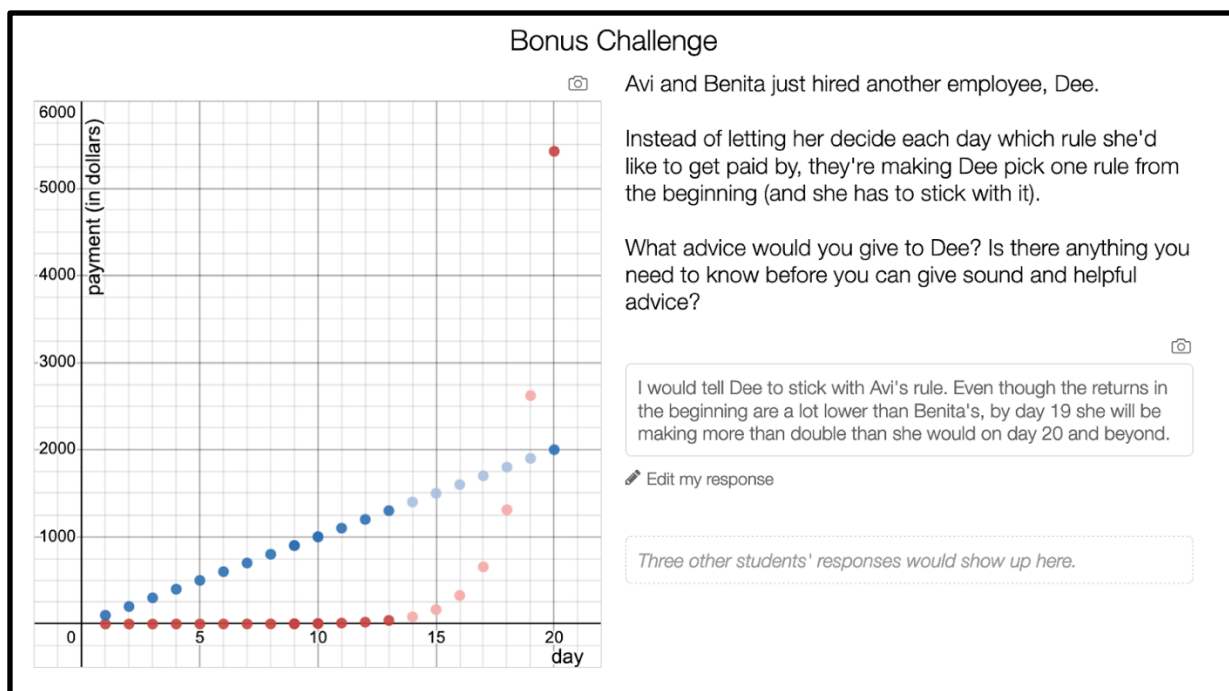
What advice would you give to Dee? Is there anything you need to know before you can give sound and helpful advice?

I would want to know what the two options they gave her were. If they were the same options I was given, I would tell Dee to pick Avi's rule from the start. It may look like someone who picked Benita's rule will make more money. It will only take about 20 days for her money to grow tremendously. She will make \$5000, \$10000, then \$20000 a day. This means that she would have more money in 3 days than someone who chose Benita's rule does the entire time.

Edit my response

Three other students' responses would show up here.

Sample Teacher Responses for Page 9



Sample Teacher Response for Page 9