🔗 Module 4 Overview Document

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	Homework	4.1 Avi & Benita's Repair Shop
		Homework	4.2 Noticing Student Thinking: Avi & Benita's Repair Shop Task
	Day 1	10 min	4.1 Discussion
		20 min	4.2 Discussion
		45 min	4.3 Selecting & Sequencing: Avi & Benita's Repair Shop
	Day 2	75 min	4.4 Three Animals Race
	Day 3	45 min	4.5 Noticing Student Thinking: Three Animals Race
		30 min	4.6 Assessing and Advancing Student Thinking: Three Animals Race

4.2 Facilitation Notes

Prior to completing this task, teachers must have first completed 4.1 Avi & Benita's Repair Shop. In this task (4.2), teachers will see students completing the same task and be asked to anticipate, attend to, and interpret students' reasoning about the ways that Avi & Benita's pay changes over time.

We recommend 4.2 to be done for homework after completing 4.1. Since teachers have completed this independently, we recommend that they first have time to discuss their responses in small groups (about 10 mins). Provide teachers with the transcripts (which are linked below the videos on the video pages provided in the 4.2 worksheet) to refer to during this discussion.

Then bring the class together to discuss responses. Two important things to make sure come out during this discussion are related to Q6 and Q8 as described below.

For Q6, we suggest discussing the exchange between Carly & Alex, during their work on Page 7 (video 4), which highlights an interesting difference in Carly's verbal explanation vs. what Alex is typing. They are noticing and explaining their understanding of the way that Avi's pay is changing over time (the rate of change) differently.

Q8 refers to instances in the video clips in which we see the students try to use the technology in ways that the task did not allow. Specifically, Elise and Fin tried to use the tool to see where the discrete graphs intersect on page 6 of the task (video 2), while Alex and Carly tried to zoom in on an image they created on page 5 of the task (video 3). This is a good opportunity to have a discussion about design choices and their impact on student thinking.

4.2 Sample Responses
Noticing Student Thinking: Avi & Benita's Repair Shop

In this task you will examine students' work on Desmos' Avi & Benita's Repair Shop Task. The task was used as an introduction to exponential functions. Specifically, it uses multiple representations (e.g., written descriptions, graphs, and tables) of linear and exponential situations to compare and contrast within the context of two different ways of being paid. The task provides a real context for the meaning of a and b ($f(x) = ax^b$), and after this task (not during), formal language regarding exponential functions will be introduced.

Thus, the overarching learning goal for this task is for students to understand the difference between linear and exponential growth in context (though they have not yet been introduced to the term "exponential"). The performance goals include that students will identify if a situation is changing constantly (i.e., common difference) or exponentially (i.e., common ratio) given a graph or table, and justify their responses using the rate of change.

🔗 [Desmos' Avi and Benita's Repair Shop Task](http://...)

Q1. On Page 5 of Avi and Benita's Repair Shop, many students say that Benita's rule will make more money on Day 20. The prompt directs the students to use math to support their answer and points to the sketch tool as one possible way to illustrate their thinking. Anticipate three different arguments that students may make to justify their answer that Benita's rule makes more money. At least one of your anticipations should include how students may use the sketch tool.

Sample Response 1:
- Benita's rule will pay more because when you follow the rate between days it increases a lot more than Avi's ever does.
- Avi is basically making little to no money and Benita is already making so much.
- The sketch of Avi graph is so close to the x axis it will take a long time to become greater than Benita.

Sample Response 2:
- They may say Benita will win because looking at the graph hers will continue to grow at a constant change.
- When drawing the sketch, they may notice that Benita's grows higher, and Avi's barely goes up at all.
- Avi's starts out being little to no money and Benita's starts out at a lot and continues to grow.

Q2. Elise and Fin say Benita's rule will make more money. Watch the video as they continue to work on page 5 of the task. Attend to (describe in detail) how the students used math and/or the technology to analyze the payment options and support their answer.

▶ [Video 1: Elise and Fin Engaging with Desmos' Avi and Benita's Repair Shop Task](#)

- Their first observation was that Benita's starting price was higher. They estimate the price at day 20 by multiplying each day by 10 to get an estimate. They make the observation that not only does Benita have a higher starting price but a steeper slope so they make the conclusion of Benita will make more money on day 20.
- They said that Benita will be at $520 on day 10 and multiplied it by two because 10 times 2 is 20. They said that Avi would make $51.20 because they multiplied her earnings by 10. The students believe that Benita would still be paying more than Avi "even if we were doing it wrong".

Q3. Continue watching as Elise and Fin work on pages 6 and 7 of the task. What prompts them to realize that Benita's rule does not pay the most on day 20? Explain in detail.

▶ [Video 2: Elise and Fin Engaging with Desmos' Avi and Benita's Repair Shop Task](#)

- They look at the dots on the graph and make the observation that they calculated Avi's graph wrong. They notice that as Benita's graph is more of a constant rate across the days, Avi has an exponential rate so it starts off small and increases rapidly closer to day 20. They notice that the graph of Avi overtakes the graph of Benita.
- They realize that they "may have done some incorrect calculations for Avi's rule" -Elise because seeing more of the graph they realize and go back and fix their graph and they realize that "The red crosses the blue a lot." Fin notices that "since it doubles the line gets steeper and steeper." They come to the conclusion that Benita's stays constant and Avis take a really sharp curve.

Q4. Based on your responses to Q2 and Q3, interpret Elise and Fin's current understanding of linear and exponential rates of change?

- Elise and Fin understand that even if the rate of change for an exponential graph starts off small, it increases more rapidly than a constant linear graph. That after a few days the exponential graph will overtake the linear graph because the slope of the line becomes more prominent.

- In the beginning they were riding the struggle bus and then they realized that the doubling that the exponential function is doing is not the same as multiplying by two. They also realized in the end that the exponential function's rate of change is more drastic than a linear and will eventually overtake any linear function it's being compared to.

Q5. Alex and Carly also say Benita's rule will make more money. Watch the video as they work on page 5 of the task. Attend to (describe in detail) how the students used math and/or the technology to analyze the payment options and support their answer.

▶ [Video 3: Alex and Carly Engaging with Desmos' Avi and Benita's Repair Shop Task](#)

- They use the line tool to see the rate of change for the blue line and red line. They use the lines to determine that Benita will pay $2000 on day 20 which is higher than Avi. They determine that Avi will not have paid over $500 by day 20 because the dots stay between 1 and 2 based off of their lines. They try to zoom in to get a more detailed look but the graph does not allow them.
- They draw straight lines for both Benita's and Avi's graph so that Benita is paying 2000 while they decide that Avi is paying some decimal amount between one and two "just not as much as Benita." Carly starts comparing them to rectangles and squares, then they attempt to zoom in to see Avis graph clearer.

Q6. Continue watching Alex and Carly as they work on pages 6 and 7 of the task. What prompts them to realize that Benita's rule does not pay the most on day 20? Explain in detail.

▶ [Video 4: Alex and Carly Engaging with Desmos' Avi and Benita's Repair Shop Task](#)

- They do the math using the table to see the rate of change from day to day for each person's rule. They see that Avi's rule gets over 5,000 while Benita's was 2,000, so they make the conclusion that Benita's rule does not pay the most on day 20.
- The table of values prompts them to rethink the rule that they came up with for Avi's graph. The big "hmm.." after the graph populated caused them to look at a graph that was not linear because the graph "does a whole lot more".

Q7. Based on your responses to Q5 and Q6, interpret Alex and Carly's current understanding of linear and exponential rates of change?

- Their understanding of linear rate of change is that it is continuous (same rate of change) throughout the days. While exponential has an increase rate of change that increases more and more as the days go on.
- I think Carly and Alex's understanding is pretty good for this introductory activity "Eventually Avi's graph is going to have bigger and bigger gaps between the two points." and they noticed that Benita's is going to "stay increasing by the same thing every time".

Q8. Both pairs of students attempt to use the technology in a way that the task did not allow.

a. What did Elise and Fin want to be able to do? How might their understanding have been affected if they had been able to do it?

- Fin and Elise wanted to be able to see how much each person paid at day 20 using the graph and graphing estimated points for where the two might intersect. Their understanding was based on a guestimate on where the payments would cross. If they had been able to connect the dots, like make a connected graph, they may have been able to draw and point and make a more exact answer.
- Fin and Elise were not able to reason correctly until they went back and changed their original table on slide 6 because they did not understand why the graph went back down. When they corrected their table they saw how Avi passed Benita "by a lot". Then they wanted to click to see where the two cross, but they couldn't do that. I am thinking that if they could see where they cross they would have figured out when that occurred more specifically.

b. What did Alex and Carly want to be able to do? How might their understanding have been affected if they had been able to do it?

- Carly and Alex wanted to be able to see the rate of change by zooming in on the graph. Their understanding changed because the graph would not allow them to zoom, so it may have seemed like both lines would be linear. But the table showed them that one was linear while the other was exponential. If they had been able to zoom in on the graph for slide 5, I am not sure how much their understanding would have changed because the 'lines' they sketched to extend through day 20 appear to both be constant even though Avi's is doubling. By day 10 the difference doubling makes is not very clear yet.
- They wanted to be able to zoom in but the graph did not allow for it. This caused them to come to the wrong conclusion that both graphs were linear. I think if they were able to zoom in they would both still appear to be linear, unless you could zoom in a lot to try to see if Avi's has started to curve by day 10.