Module 4 Overview Document

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	Homework	4.1 Avi & Benita's Repair Shop
		Homework	4.2 Noticing Student Thinking: Avi & Benita's Repair Shop
	Day 1	10 min	4.1 Discussion
		20 min	4.2 Discussion
		45 min	4.3 Selecting & Sequencing: Avi & Benita's Repair Shop
	Day 2	75 min	4.4 Three Animals Race
	Day 3	45 min	4.5 Noticing Student Thinking: Three Animals Race
		30 min	4.6 Assessing and Advancing Student Thinking: Three Animals Race

4.5 Facilitation Notes

The videos included in this task were recorded with high school students completing the 4.4 Three Animals Race Task using a slightly different GeoGebra file. Rather than having separate GeoGebra files linked throughout the task, the students' version collected the files into a GeoGebra book. The Three Animals Race GeoGebra Book provides secondary students with four pages that include the simulation (Page 1), the spreadsheet (Page 2), a STOP page (Page 3), and a page that includes a correct, completed spreadsheet and graph view (Page 4).

Three Animals Race Task

The task document, *4.5 Noticing Student Thinking: Three Animals Race* needs to be shared electronically with teachers as it has hyperlinks to videos within.

In this activity, teachers will examine authentic student work on the task. They will see two different pairs of students, each working collaboratively on one laptop. They will see the students' computer screen (i.e., their technology engagement) and listen to them discuss their ideas. Specifically, these video clips are focused on the ways that students use different technological representations (e.g., simulation applet, spreadsheet) to make sense of the animals' rates of change. Teachers will attend to students' reasoning and interpret students' current understanding of rate of change, with a focus on how technology facilitates their understanding (or not). Sample responses are included below.

We recommend completing 4.5 Noticing Student Thinking: Three Animals Race in small groups during class watching the videos and discussing their thoughts along the way.

This should be followed by a full class discussion focused on Q6 and Q7. Instructors might want to remind teachers to bring their airpods/headphones to class to aid in listening to the videos.

Sample Responses (pp. 2–7) are below.

4.5 Sample Responses
Noticing Student Thinking: Three Animals Race

High school students engaged in the Three Animals Race Task. The task was used as an introduction to describe different types of rates of change. Specifically, it was intended to address the following learning and performance goals:

- Students will understand that not all contexts can be described as changing linearly or exponentially.
- Students will explain the difference between the three animals in the race using their rates of change. Specifically, the turtle is moving away from the starting line at a constant rate (linear), rabbit is moving away from the starting line at a constantly additive rate (quadratic), and alligator, after holding still for a while, is moving away from the starting line at a constantly multiplicative rate (exponential).
- Students will model the race and determine a winner by building a table in a spreadsheet using what they know about the animals' rates of change and distance from the starting line.

Three Animals Race Task

In this video clip you are going to see Fiona and McKenzi working on the Three Animals Race task. The clip begins with their prediction of who will win the race and then you will watch portions of their work as they use the simulation and spreadsheet to determine a winner of the race. While watching the video, focus on the students' language, what they do with the technology, and what they see as a result of what they do.

Fiona & McKenzi Engaging with the Three Animals Race Task

Q1. Attend to how Fiona and McKenzi reasoned about Turtle, Rabbit, and Alligator's distance from the starting line during the race.

- The turtle was at a constant speed, he was slow, so they assume he is linear. The rabbit after every second increased speed by 1 meter per second, assuming quadratic. The alligator stayed at the same speed until 6 meters and then increased in speed drastically, so they assume he is exponential.

- They noticed that it was going to be an exponential line for the rabbit and the alligator, they determined that either the rabbit or the alligator was going to win but they were not sure yet with just the racing simulation. When filing out the chart at the beginning they did not read the instructions correctly.

- The girls initially say the turtle will win because linear will get there faster. They also discuss that exponential will curve up.

  When they watch the simulation they say that alligator wins and discuss that rabbit starts out exponential, but that alligator is also exponential but starts out less steep but then gets steeper after 5 seconds.

  They quickly fill out the table for the turtle's distance by counting by three for each second, but they forget that he started at 50, not zero. For rabbit they initially complete the table with the rate of change for each second, not the distance from the start line. They do the same thing when completing the spreadsheet for alligator, record her rate of change for each second. Instead of multiplying her rate by 1.5 each second, they add 1.5 each second. Basically, using the same reasoning as they did with rabbit.

Q2. Interpret the students' current understanding of the rate of change for each of Turtle, Rabbit, and Alligator. Provide evidence from the video (including both what students verbalize and the way they engage with the technology) to support your claims.

- Their understanding is that if the rate of change is constant it is linear. If the rate of change increases by multiplying by two, it is quadratic. If the rate of change is the same and then increases really fast (drastically) it is exponential.

- They understand that the turtle goes up 3 each time and that will be a linear line, for the rabbit they figured out that it goes up one more meter each time it goes up.

- They understand that turtle runs the race at a constant rate of 3 m per second. They initially filled out the spreadsheet with him starting at zero, but when they began to look at rabbit, they realized they forgot he started at 50 m. They fix that and are able to complete the spreadsheet by adding 3 m to her distance each second.

  When the students first discuss rabbit's rate of change they clearly understand that her rate of change goes up by one each time and that is what they record in the spreadsheet, instead of her distance from the starting line. Later the video shows that they realized that the values they recorded were her rate of change each second but they needed to add the distance he traveled based on the rate

of change to get the distance from the start. The video shows a partially completed table with Fiona explaining it to McKenzi, then McKenzi asks Fiona to show her because she is visual. They use the spreadsheet to start to fill in the distances while they are talking about the increase of one in the rate of change for each second. When the numbers begin to get larger they use the spreadsheet to calculate the values.

We do not see them talk about alligator, but the last part of the video shows they changed their values for alligator, and they say they are going to correct hers after they finish rabbit. From the values in the table they appear to still think they need to add 1.5 m/sec to get alligator's rate at each second, but they do understand that her rate is increasing each second and that they have to use that to figure out the distance from the start line. I wonder if they eventually realize that her rate is multiplied by 1.5 m/sec, not add 1.5 m/sec.

Q3. In what ways did the technology tools the students used (e.g., the simulation applet, the spreadsheet tools) inform (or not) the students' thinking about the race? Explain.

- The simulation allowed the students to visualize the speed of each animal throughout the race. They could see the turtle was moving at a constant speed, the alligator stay and then go really fast. Then they were able to use the spreadsheet to make mathematical conclusions on how long it took each animal to finish the race using rate of change for each.

- The simulation helped them to understand that they made a mistake on the chart because they had the turtle beating the rabbit in the chart. Once they got the numbers right, they filled the chart out correctly.

- The simulation helped the students see that their initial guess that turtle would win was incorrect. They concluded that alligator won based on the simulation.

  The spreadsheet helped the students examine each animals' rate of change by one second intervals. They easily added three meters each second for turtles' distance, initially and later when they realized the starting distance error.

  The spreadsheet also helped the students initially record their understanding of each animals' rate of change. Although they started by recording rabbit and alligator's rate of change (their reasoning - add one each second and add 1.5 each second, respectively), they eventually used this information to realize they had to add the distance traveled each second to the previous value to get the distance from the start line. The video shows that they were able to use the computation capabilities of the spreadsheet to help them do this with rabbit.

The one issue with the spreadsheet (that the students were quick to realize) was that the numbers indicating the row number of the spreadsheet were one off of the seconds elapsed. I could see this confusing some students.

Next, watch Gibson and Will as they reason about the Three Animals Race. Like Fiona and McKenzi, the clip begins with their prediction of who will win the race and then you will watch portions of their work as they use the simulation and spreadsheet to determine a winner of the race. While watching the video, focus on the students' language, what they do with the technology, and what they see as a result of what they do.

Gibson & Will Engaging with the Three Animals Race Task

Q4. Attend to how Gibson and Will reasoned about Turtle, Rabbit, and Alligator's distance from the starting line during the race.

- Before playing the simulation Will thinks the Alligator will win because he will speed up exponentially. Gibson is debating because he initially thought Turtle would will win since he had a 50 m head start, and it would only take him about 15 seconds. He has second thoughts about that because Alligator is running at an exponential rate.

- The turtle would win the race because he started at 50 meters and they guessed that it would take him 15 seconds to reach the finish line. They also state the turtle and rabbit go at a constant rate while the alligator's speed is being multiplied.

- When working on Turtle, at first Gibson puts 50 m at 1 sec, and will explains how he has traveled 3 m in the first second. Gibson then says 47 m because he is thinking about distance from the finish line, but they correct themselves and say it is distance from the start. Then they fill out the spreadsheet for Turtle in increments of 3 m per second.

  When working on Rabbit, they start out by saying her rate is exponential, but that turtle is linear. They start to fill in the spreadsheet with 1, 3, 6. At first Will explains this with the wrong unit 2 seconds, but then corrects the units to say 3 m which adds to 6, then you will add four, so it is going to be some kind of exponential. Then Gibson lets Will fill in the table. He does this by adding one more to the distance each interval to get the next distance.

  When working on Alligator, they record 0 for the first five seconds, then 1.5 m for 6 seconds because her starting rate was 1.5 m per sec. There is some confusion and discussion about the next value. Gibson says it is 1.5 to the x, but Will explains that he calculated 1.5 times 1.5 then added it to the last position. Gibson is thinking it is 1.5 to the x each time, but he says he is confusing himself. Will

then explains how he got 3.75 m for 7 seconds, which was to do 1.5 times 1.5 because that is how Alligator speeds up, but then add that to last position. The teacher asks what the next one would be, and after thinking Will says to take 2.25 (1.5 * 1.5) and multiply that by 1.5 again and add 3.75 = 7.125. Gibson asks what you are adding back and realizes he is adding back the original. They continue to multiply the last rate by 1.5 and add back the last value to complete the spreadsheet.

Q5. Interpret the students' current understanding of the rate of change for each of Turtle, Rabbit, and Alligator. Provide evidence from the video to support your claims.

- Turtle had a constant rate of change throughout the entire race. Rabbit has a rate of change where it would be multiplied by two throughout the race. Alligator goes 1.5 to the x power, and add the previous amount to the speed to get where the alligator is at during the race.

- They understand that the turtle moves 3 meters every second, the rabbit moves at a meter faster than the previous time, 1, 3, 6, which they believe is an exponential growth until the graphing calculator gives them an error. They understand that alligator is exponential, multiplied by 1.5 each time.

- By the end of the video, they specifically state that Rabbit is linear with a y-int of 50. They say that Rabbit is quadratic, but that conclusion comes from Will trying to do the exponential regression and getting an error, not reasoning about the change in Rabbit's rate by adding one each second. They have stated the whole time that Alligator was exponential, but I did not see where they connected that to the fact you multiply by 1.5 each time to get the new rate. They knew how to compute each rate and distance accurately, but I did not see in the video where they connected the rate of change to the fact that linear is constant, quadratic is additive, and exponential is multiplicative.

Q6. In what ways did the technology tools the students used (e.g., the simulation applet, the spreadsheet tools, the graphing calculator tools) inform (or not) the students' thinking about the race? Explain.

- The simulation allowed them to reason about the hypothesis of how each animal would move during the race. The spreadsheet allowed the students to write down where the animal would be during the race and compare it to one another to see which animal had the highest rate of change.

- They were able to understand how each of the movements from the animals were calculated, for the alligator they used a calculator to figure out how fast it moved.

- Once they play the simulation they conclude that Rabbit and Alligator finish between 13 & 14 seconds. The simulation helped them see that either Rabbit or Alligator won, not Turtle.

  They did not really use the spreadsheet to compute any of the distances for each animal. I think the spreadsheet really just served as a recording tool.

  They did all the calculations on the graphing calculator. When the teacher asks them to predict what the graph of each would look like. They say linear for Turtle, with a y-int of 50. Will wants to find the regression lines for each. They say exponential for Rabbit and Alligator. The video shows the work Will did to find regression for Rabbit, but it is quadratic, and he says exponential gave him an error and quadratic gave him an r of 1. The graphing calculator helped them do the regression and realize that Rabbit was not exponential due to getting an error. The video cuts off here, so I wonder what happens to their thinking about Rabbit being quadratic, not exponential. Do they figure out what makes her rate quadratic other than the graphing calculator giving them an error?

Q7. We often focus on equations and graphs when modeling situations mathematically. Here, the students were "forced" to use tables as one of the animals could not easily be modeled using a function that they are familiar with (i.e., alligator). In what ways might focusing on tables to represent the situation prior to creating graphs have supported their thinking about rate of change? Explain.

- I think it helps them see that the numbers are increasing each time and how they are increasing. It gives them specific numbers where if you were looking at a graph it may be more difficult to see which one is which.

- Focusing on the tables allows the students to actually calculate the rate of change and not just visualize it like a graph would. This allowed both groups of students to see how each type of graph drastically changes throughout the time that was recorded.

- When the students reasoned from the table they had to think about and find the rate of change at each second to find out how far each animal traveled in each one second interval, then use that to compute the distance from the starting line. If they had been given a graph first, they would have seen the total distance traveled (what they computed in the table), but not explicitly have to reason about what the rate of change was for the distance to increase according to the rule for

each animal. The table 'forces' them to actually find each rate of change, whereas the graph does not.