Module 4 Overview Document

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	Homework	4.1 Avi & Benita's Repair Shop
		Homework	4.2 Noticing Student Thinking: Avi & Benita's Repair Shop
	Day 1	10 min	4.1 Discussion
		20 min	4.2 Discussion
		45 min	4.3 Selecting & Sequencing: Avi & Benita's Repair Shop
	Day 2	75 min	4.4 Three Animals Race
	Day 3	45 min	4.5 Noticing Student Thinking: Three Animals Race
		30 min	4.6 Assessing and Advancing Student Thinking: Three Animals Race

4.4 Facilitation Notes

Provide teachers with the digital worksheet that includes the applet links, or if using a printed copy of the task, you will need to provide the applet links separately.

Three Animals Race Task

We recommend launching this task using a brief video of the race teachers are familiar with, the Disney Classic Tortoise and the Hare. We suggest beginning the clip around 2:10 and showing approximately 2 minutes of the race. You can then ask, "Who do you think will win? Why?" This acts as a nice hook for the task and allows for the introduction of "alligator" to the race.

Tortoise and Hare

If teachers are not familiar with graphing polylines in GeoGebra, it is important to provide instructions for how to do so. This information may be found in the PTMT: Algebra Materials in Chapter 1: Introduction to Technological Representations, Section 3 (pp. 9–10).

It is important to point out to teachers upfront that they are *not* expected (nor should they try) to determine the function to represent each animal's race. The focus should be on building up the animals' distance from the starting line at each second in the race based on what you know of their rate of change.

Additional Task Commentary (p. 2) and Sample Responses (pp. 3–7) are below.

4.4 Commentary
Three Animals Race Task

We suggest the 4.4 Three Animals Race Task be completed in class with teachers working in small groups and discussing their thinking while completing the task. Particular aspects to attend to and discuss as they complete the task are their initial predictions, their reasoning about each rate of change, their final conclusions, and how their use of the spreadsheet and graph facilitated their thinking about each animal's rate of change. Once they have completed the task there should be a discussion about how they anticipate high school students might approach the same task.

When being asked to make sense of either contextual situations or data they hope to model, teachers need to be able to recognize and describe the rate of change both clearly and concisely. This should have been included as part of the discussion of 4.1, and this will also play a critical role in 4.5 Noticing Student Thinking: Three Animals Task. With that in mind, it is important to specifically discuss:

- When each animal begins to move and how that is represented in the table. For example, Alligator starts moving at 5 seconds which means she is still at the start when the time hits 5 sec, but at 6 seconds her distance from the start is 1.5 m.
- How to best (i.e., clearly) describe how each animal's speed is changing throughout the race.
    - Turtle's rate is constant—it is not changing
    - Rabbit's rate is constantly changing—constantly changing amounts of change
    - Alligator's rate is a constant multiplicative change—it is speeding up by multiplying rate by the same amount each second
- How could you precisely describe linear, quadratic, and exponential rates of change regardless of context?
    - Linear—for equal changes in x, the dependent variable changes by a constant amount
    - Quadratic—for equal changes in x, changes in the dependent variable increases/decreases by the same amount
    - Exponential—for equal changes in x, the dependent variable is multiplied by the same amount.

In addition to discussing specific answers to the questions posed in the task, to push towards thinking about the role of the technology in the sensemaking of this task, pose the following:

- How did being forced to use tables/spreadsheets help you make sense of each animal's rate of change?
- How did using the simulation support your sensemaking about the race?
- What did creating the graph add to your thinking that was different from the simulation and spreadsheet tables?

4.4 Sample Responses
Three Animals Race Task

This task explores the infamous race of the tortoise and the hare. Well, there is another version of the story that includes an alligator! It goes something like this:

Turtle, Rabbit, and Alligator are competing in a 100-meter dash. Rabbit is excited, she knows she is the fastest of them all. She's bouncing around at the starting line taunting Turtle. Alligator and Rabbit decide to give Turtle a 50-meter head start. So the race referee has Turtle line up 50 meters ahead of Rabbit and Alligator when she announces, "Take your mark...get set...go!"



Rabbit takes off slowly but keeps getting faster, starting at a rate of 1 meter/sec and going 1 meter/sec faster each second of the race until she arrives at the finish line. Turtle takes off and walks at a steady rate of 3 meters/sec for the duration of the race. When the starting signal sounds, Alligator is caught off guard. She stands still and watches Rabbit and Turtle take off. At five seconds she finally starts running. She begins at a pace of 1.5 meters/sec and continues to speed up, going 1.5 times faster each second of the race.

The story ends with a winner of the race, but who is it?

Q1. Make a prediction. Who wins?

- Turtle will win because of the head start the others will not be able to catch up.
- Rabbit will win because she increases her speed by 1 meter each second, so she speeds up to cross the finish line first.
- Alligator will win because even though she gets startled, she increases her speed 1.5 times faster each second. This will help her catch the others and finish first.

Let's explore the race using a simulation.

Three Animals Race Task

On the first page of the GeoGebra book, drag the *time* slider to observe a simulation of the race.

Q2. Based on what you have observed, who do you think wins the race and why?

- Rabbit wins because she is closer the the finish line at 13 seconds than alligator.
- Alligator wins because even though she is further away from the finish line at 13
- seconds, she is further past the finish line at 14 seconds. This means she will catch and pass rabbit to win.

Let's use a spreadsheet to create a model of the race.

Three Animals Race Spreadsheet

Examine the spreadsheet set up. Notice the column on the left indicates the amount of time that has passed since the race started. The next three columns are set up to record the distance from the starting line each animal is at each second since the race began.

Q3. How many meters has each animal traveled after 0 seconds? 1 second? 2 seconds? 3 seconds? Record these in the spreadsheet. Include a screenshot of your spreadsheet.

	A	B	C	D
1	time (sec)	Turtle Distance from Start	Rabbit Distance from Start	Alligator Distance from Start
2	1	53	1	0
3	2	56	3	0
4	3	59	6	0

Figure 1: Sample Teacher Response to Q3

Q4. In your own words, explain how each of Turtle, Rabbit, and Alligator's rates change throughout the race.

Turtle: Turtle just keeps going 3 meters each second, so her rate remains the same/constant/doesn't change throughout the race.

Rabbit: Rabbit speeds up one meter each second, so if she went 1 m/sec until the first second, then she would go 2 m/sec from 1-2 seconds, 3 m/sec from 2-3 seconds, and so on. Just add one m/sec each additional second.

Alligator: Alligator has no rate of change for the first five seconds because she got startled. Then from 5-6 sec she runs at 1.5 m/sec, but she speeds up 1.5 times what she was running each second before. That means her rate for the

previous second has to be multiplied by 1.5 for each additional second. So from 6-7 seconds that would be 1.5 [rate from 5-6 seconds] * 1.5 [increase in rate for 6-7 seconds], then from 7-8 seconds (1.5 * 1.5) [rate from 6-7 seconds] * 1.5 [increase in rate for 7-8 seconds] and so on.

Q5. In your spreadsheet, continue building up the animals' distances from the starting line as the race continues. Approximately how many seconds does it take for each animal to finish the race? Include a screenshot of your spreadsheet.

	A	B	C	D
1	time (sec)	Turtle Distance from Start	Rabbit Distance from Start	Alligator Distance from Start
2	1	53	1	0
3	2	56	3	0
4	3	59	6	0
5	4	62	10	0
6	5	65	15	0
7	6	68	21	1.5
8	7	71	28	3.75
9	8	74	36	7.13
10	9	77	45	12.19
11	10	80	55	19.78
12	11	83	66	31.17
13	12	86	78	48.26
14	13	89	91	73.89
15	14	92	105	112.33
16	15	95	120	170
17	16	98	136	256.5
18	17	101	153	386.25
19	18	104	171	580.87

Figure 2: Sample Teacher Response to Q5

Turtle: Between 16-17 seconds

Rabbit: Between 13-14 seconds

Alligator: Between 13-14 seconds

Q6. Rabbit and Alligator both sped up during the race, whose rate of change is increasing faster? Explain.

- Alligator because her rate is multiplied each time. Rabbit adds each time. Multiplying increases the rate faster than adding.

Q7. Who wins the race? How do you know?

- Alligator because her rate between 13-14 seconds is 1.5^9 m/sec which is faster than Rabbit whose rate is 14 m/sec during the same time. This means she will pass Rabbit to win.
- Rabbit will win. Even though Alligator speeds up, Rabbit is closer to the finish line at 13 seconds and will cross the finish line first.

Q8. If you were to create a graph of time vs. distance from the start line for each of the animals, what would you expect each graph to look like? Explain.

Turtle: a line that starts at 50 m on the y-axis because she got a head start and her rate of change is a constant 3 m/sec for the race

Rabbit: starts at zero/starting line then curved and getting steeper each second since her rate is going up by 1 m/sec each interval

Alligator: starts at zero and stays there for the first five seconds, so horizontal line; then curved and getting steeper faster than Rabbit since Alligator's rate is multiplied by 1.5 m/sec each interval

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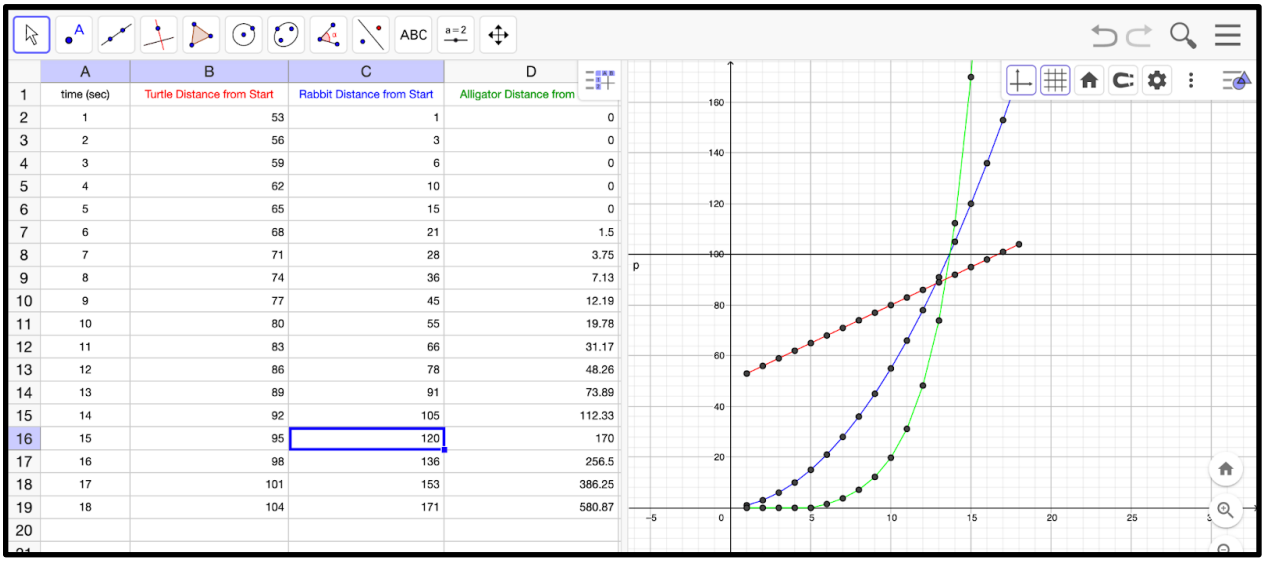
Add a graphing view and use the create polyline tool to graph each animal's distance from the starting line over time. Please include a screenshot of your graph.

Note: To select two columns of data that are not adjacent to each other, select one and then hold down the "control" key to select the second.



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9	8	74	36	7.13
10	9	77	45	12.19
11	10	80	55	19.78
12	11	83	66	31.17
13	12	86	78	48.26
14	13	89	91	73.89
15	14	92	105	112.33
16	15	95	120	170
17	16	98	136	256.5
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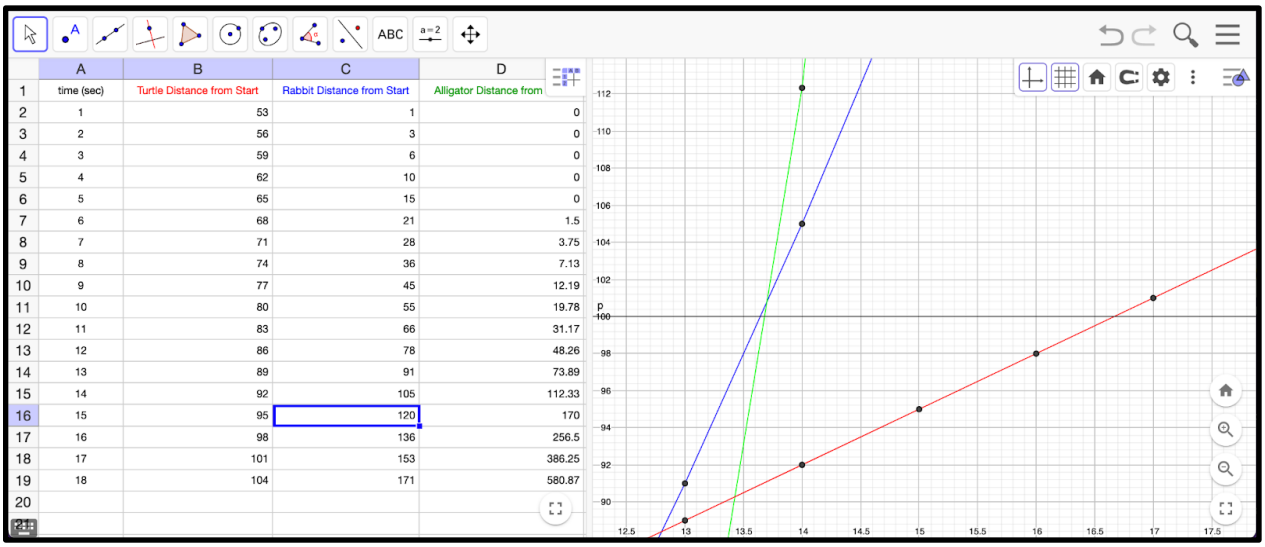
Figure 3: Sample Teacher Response to Q9

Note: The graphic in Figure 3 used the Algebra view to change the color of each polygraph to match spreadsheet headings, and added $y = 100$ to help reason about when each animal will cross the finish line. Zooming in on the Graphics view may assist in this process (Figure 4).



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17	16	98	136	256.5
18	17	101	153	386.25
19	18	104	171	580.87
20				

Figure 4: Sample Teacher Response to Q9

Q9. Approximately how long did it take for each of the contestants to finish the race? How do you know? Who won?

- Turtle finished at approximately 16.65 seconds because that is where the graph of her distance crosses the 100 m mark. She finished third

- Rabbit finished at approximately 13.65 seconds because that is where the graph of her distance crosses the 100 m mark. She won.
- Alligator finished at approximately 13.7 seconds because that is where the graph of her distance crosses the 100 m mark. She finished second, by a hare...LOL!

Q10. Who would have won if the race were 95 meters rather than 100? Why?

- Still Rabbit because they (the blue line) reach 95 m before Alligator (the red line)

Q11. Who would have won if the race were 115 meters rather than 100? Why?

- Alligator would win because they (the green line) reach 115 m before Rabbit.