



Comparing and Contrasting Linear, Quadratic, and Exponential Rate of Change

Total time: 225 minutes but could be shortened to 75 minutes if needed. See page 2 for recommendations for a 75-minute module.

Situating the Module within the PTMT Teaching Algebra Materials: This module could be used alongside Chapter 5: Rates of Change of the PTMT Teaching Algebra Materials. Section 2 of the chapter, *Representing and Comparing Rates of Change*, is focused on leveraging teachers' experiences with linear, quadratic, and exponential rates of change. If using both sets of materials, 4.4 Three Animals Race is the same task in each set of materials.

Overview of the Module: This module focuses on comparing and contrasting linear, quadratic, and exponential rates of change. The work is situated in two different mathematical tasks; one is an exploration of a linear and exponential payment structure, and one is a dynamic representation of a race between three animals whose rates are different (linear, quadratic and exponential). For each task, teachers will engage with the task, anticipate student responses, and attend to and interpret student thinking. For the payment task, teachers consider students' engagement as students compare the compensation structures, and they use Desmos teacher tools alongside a pre-populated teacher dashboard to select, sequence, and plan for a whole class discussion. For the race tasks, teachers consider students' engagement as they move across dynamic representations of the animals' movement through tables and graphical representations of their distance vs. time, and they focus on assessing and advancing students' thinking through posing purposeful questions.

Module Goals:

- Consider how to use math action technologies focused on multiple representations for students to explore varying rates of change
- Examine student practices and practice anticipating, attending and interpreting student thinking on technology-enhanced tasks exploring varying rates of change
- Engage in selecting and sequencing student work within Desmos Teacher Dashboard
- Practice posing purposeful questions to students as they work within technology-mediated environments



Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	Homework	4.1 Avi & Benita's Repair Shop
			4.2 Noticing Student Thinking: Avi & Benita's Repair Shop
	Day 1	10 min	4.1 Discussion
		20 min	4.2 Discussion
		45 min	4.3 Selecting & Sequencing: Avi & Benita's Repair Shop
	Day 2	75 min	4.4 Three Animals Race
	Day 3	45 min	4.5 Noticing Student Thinking: Three Animals Race
		30 min	4.6 Assessing and Advancing Student Thinking: Three Animals Race

Recommendations for 75-minute module: This module is designed for 225 minutes but could be shortened to 75 by choosing one of the two technology tasks for teacher investigation based on the teacher practices associated with the tasks. If teachers would benefit from practicing select and sequence using Desmos teacher tools, have teachers work with Avi & Benita's Repair Shop and associated student work. If teachers would benefit from considering whole class discussions focused on multiple representations for exploring various rates of change, have teachers work with the GeoGebra Three Animals Race Task and associated student videos.

MTE preparation for Module 4:

- Engage with both technology-enhanced tasks: Avi & Benita's Repair Shop and Three Animals Race
- Read facilitation notes and commentaries, engage with assignments (including watching associated videos), and look over sample responses.
- For a deeper dive into the mathematical ideas, see pages 36–37, 42–46, 53–55 and 78–83 from Cooney, T. J., Beckmann, S., Lloyd, G. M., & Wilson, P. S. (2010). *Developing essential understanding of functions for teaching mathematics in grades 9-12*. National Council of Teachers of Mathematics.






Suggested Readings for Teachers:

- Mojica, G., & Hollebrands, K.F. (2016). The use of the five practices to support classroom discussions of technology-enhanced mathematics tasks. Teaching Mathematics with Technology MOOC-Ed, Friday Institute for Educational Innovation: NC State University, Raleigh, NC. https://fi-courses.s3.amazonaws.com/tmt/unit_4/TheUseoftheFivePracticestoSupportClassroomDiscussionsofTechnology-BasedMathematicsTasks.pdf
- Lo, J. J., & Kraky, J. L. (2012). Looking for connections between linear and exponential functions. *Mathematics Teacher*, 106(4), 295–301.






Detailed Agenda for Module 4





Table 2: Agenda

	Description of Module Tasks	Facilitation Notes
Day 0 / Homework	4.1 Avi & Benita's Repair Shop  Teacher devices  Desmos Task Teachers complete the Desmos task that has them compare a linear and exponential payment structure.	Create a class code for the Desmos Task and provide the link to your teachers. See the full Instructor Materials which include the facilitation notes and sample responses.
	4.2 Noticing Student Thinking: Avi & Benita's Repair Shop Task  Teacher devices  Desmos Task  Worksheet Teachers anticipate student responses and then notice two pairs of students' mathematical thinking as they work through various pages of the Desmos task.	Provide a copy of the electronic worksheet to the teachers. Depending on teachers' experiences in noticing in technology-mediated environments, we recommend pointing teachers to the NITE framework and reminding teachers that when noticing student thinking with technology, they should pay careful attention to what students say, do with the technology, and what they record. See the full Instructor Materials which include the facilitation notes and sample responses.
Day 1	4.1 Discussion (10 minutes)	We recommend showing the Desmos Teacher Dashboard with the teachers' responses from their homework and then utilizing the snapshot and presentation tools to model use of the Teacher Dashboard while facilitating a brief discussion of the task. See the full Instructor Materials which include the facilitation notes and sample responses.



Day 1 (cont.)	4.2 Discussion (20 minutes)	<p>We suggest putting teachers in small groups to discuss their responses prior to opening up for a whole class discussion. We recommend first highlighting the teachers' noticing with a specific focus on Q3 and Q8. It may help to provide teachers with the transcripts (linked below the video on the video page) to refer to during this discussion.</p> <p>See the full Instructor Materials which include the facilitation notes and sample responses.</p>
	4.3 Selecting & Sequencing: Avi & Benita's Repair Shop (45 minutes)  Teacher devices Teachers use a Desmos Teacher Dashboard to select and sequence student responses for a class discussion.	<p>Unlike other module tasks, there is no associated worksheet for the teachers. Instead, you will illustrate features of the teacher tools associated with teacher.desmos.com and provide teachers with an opportunity to practice selecting and sequencing student responses on pages 5 and 7 of the Desmos task. It is imperative that you read the Facilitation Notes in depth prior to facilitating this task.</p> <p>See the full Instructor Materials which include the facilitation notes and sample responses.</p>
Day 2	4.4 Three Animals Race (75 minutes)  Teacher devices  Worksheet Teachers engage in a GeoGebra task examining and comparing three different rates of change.	<p>Provide a copy of the electronic worksheet to the teachers. We recommend teachers work in small groups, so they can discuss their thinking while completing the task. As teachers work, you may want to have small or whole group discussions about their initial predictions, their reasoning about each rate of change, their final conclusions, and how their use of the spreadsheet and graph facilitated their thinking about each animal's rate of change. Once they have completed the task, you should facilitate a whole class discussion about how they anticipate high school students might approach the same task. Let the teachers know they will watch videos of high school students engaging with the task in the next class.</p> <p>Notes:</p> <ul style="list-style-type: none"> • Ensure teachers understand they are <u>not</u> expected (nor should they try) to determine the function to represent each animals' race. The focus should be on building up the animals' distance from the starting line at each second in the race based on what is known regarding their rate of change. • The Facilitation Notes include a suggestion for a video to launch the task and additional materials for teachers who need instruction on how to graph polylines in GeoGebra prior to beginning the task. <p>See the full Instructor Materials which include the facilitation notes and sample responses.</p>



Day 3	4.5 Noticing Student Thinking: Three Animals Race (45 minutes)	<p>Prior to assigning this task, let teachers know the videos included in this task were recorded with high school students using a slightly different GeoGebra file. See details in the Facilitation Notes.</p>
	<p>  Teacher devices  Worksheet </p> <p>Teachers notice two pairs of students' thinking related to rate of change and how each pair of students use different technology representations to make sense of the different rates of change.</p>	<p>Provide a copy of the electronic worksheet to the teachers. We recommend teachers completing this task in small groups so encourage teachers to use airpods/headphones to aid in listening to the videos. Have teachers discuss their responses after each video followed by a full class discussion focused on Q6 and Q7.</p> <p>See the full Instructor Materials which include the facilitation notes and sample responses.</p>
	4.6 Assessing and Advancing Student Thinking: Three Animals Race (30 minutes)	<p>Provide a copy of the electronic worksheet to the teachers. We recommend teachers complete this task as a jigsaw. Encourage teachers to keep the task open in case they want to model what the students were doing to obtain their output. When facilitation the teachers' comparisons of their noticing of the three student pairs, it is important to ensure teachers are discussing how students are interpreting starting distances, common differences and/or ratios, and how they are building their tables.</p>
	<p>  Teacher devices  Worksheet </p> <p>Teachers engage in a jigsaw task in which they attend and interpret three different pairs of students' written work and decide on assessing and advancing questions they would ask the students.</p>	<p>See the full Instructor Materials which include the facilitation notes and sample responses.</p>