



## Module 4 Overview Document

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	Homework	4.1 Avi & Benita's Repair Shop
		Homework	4.2 Noticing Student Thinking: Avi & Benita's Repair Shop
	Day 1	10 min	4.1 Discussion
		20 min	4.2 Discussion
		45 min	4.3 Selecting & Sequencing: Avi & Benita's Repair Shop
	Day 2	75 min	4.4 Three Animals Race
	Day 3	45 min	4.5 Noticing Student Thinking: Three Animals Race
		30 min	4.6 Assessing and Advancing Student Thinking: Three Animals Race

### 4.3 Facilitation Notes

In this task, teachers will examine students' work on Avi & Benita's Repair Shop Task using a prepopulated teacher dashboard to practice orchestrating a full class discussion. To be prepared for this task, teachers must have already completed 4.1.

**Note: There is no worksheet to go with this task; this is intended to be completed collaboratively in a whole class setting.**

An Overview of the task (pp. 1–2), Additional Task Commentary (pp. 3–5), and Sample Responses (pp. 6–9) are below.

## Overview

### 4.3 Selecting and Sequencing: Avi & Benita's Repair Shop

As a reminder, the overarching learning goal for this task:

- Students will understand the difference between linear and exponential growth in context (though they have not yet been introduced to the term “exponential”).

Specific performance goals include:

- Students will identify if a situation is changing constantly (i.e., common difference) or exponentially (i.e., common ratio) given a graph or table.
- Students will justify their responses using the rate of change.



## Avi & Benita's Repair Shop



With the context in mind, you will now revisit the Avi & Benita's Repair Shop Task through the lens of a teacher. First, look through the task using the student preview screens (Figure 1) with your teachers. At the bottom of each screen, Desmos provides teacher moves and sample responses. The “teacher moves” often suggest particular screens that are important to discuss and even offer suggestions for what to focus on when selecting student responses.

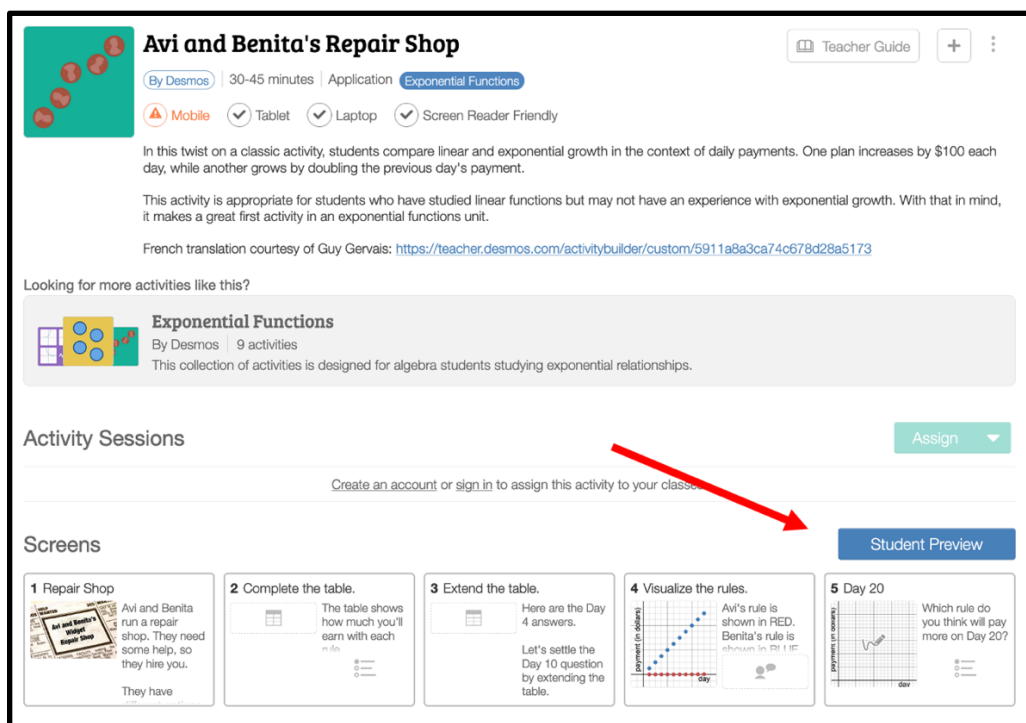


Figure 1: Where to find “Student Preview” in the Task

For this particular task, they suggest discussing pages 5 and 7 specifically.

Page 5: Highlight several student responses for the class. Start with informal math language and reasoning, then move to more formal responses.

Page 7: Highlight unique answers for the class, using them as the spark for a brief discussion about the behavior of linear (Benita's rule) vs exponential (Avi's rule) growth. It's up to you whether to introduce the term "exponential" at this stage. You might simply call it "nonlinear" for now.

The link below will take you and your teachers to your own teacher dashboard with responses populated from an actual Integrated Math 1 class. Teachers will use this sample dashboard, the snapshot tool, and present mode to practice selecting and sequencing students' responses for a class discussion focused on pages 5 and 7 of Avi & Benita's Repair Shop.



### [Sample Teacher Dashboard for Avi & Benita's Repair Shop](#)

You will ask your teachers to answer Q1 and Q2. Teachers will use the Teacher Dashboard's snapshot tool and present mode to select and sequence students' responses. For each of page 5 and page 7, teachers will select between 1 and 4 snapshots, sequence them, and press the present button before taking a screenshot. Teachers will use the Teacher Move suggestions to guide their selections.

**Q1.** Present the students' responses you selected and sequenced for a class discussion for **page 5** of the task. Explain your select and sequence strategy.

**Q2.** Present the students' responses you selected and sequenced for a class discussion for **page 7** of the task. Explain your select and sequence strategy.

## 4.3 Commentary

### Selecting and Sequencing: Avi & Benita's Repair Shop

If teachers are not familiar with Smith & Stein's (2011) Five Practices for Orchestrating Productive Mathematics Discussions (selecting and sequencing in particular), then we recommend reading and discussing the following prior to engaging with the task.

- Chapter 1 of Smith, M. S., & Stein, M. (2018). *Five practices for orchestrating productive mathematics discussions*. NCTM.
- Mojica, G. & Hollebrands, K. F. (2016). The use of the five practices to support classroom discussions of technology-enhanced mathematics tasks. Teaching mathematics with technology MOOC-Ed, Friday Institute for Educational Innovation: NC State University, Raleigh, NC.

In addition, if teachers are not familiar with the Desmos Teacher Dashboard, we recommend providing an overview before beginning the task. A thorough introduction is provided in Module 3 (Task 3.4), and additional support can be found at [learn.desmos.com](http://learn.desmos.com). If this is the first time teachers are using the teacher dashboard, they may need additional support using the snapshot tool.

Once teachers are familiar with the 5 Practices and the Desmos Teacher Dashboard, they are ready to start 4.3 Selecting & Sequencing Student Thinking on Avi & Benita's Repair Shop. They will need to be logged into Desmos to access the dashboard in the task. We suggest doing this task during class so that teachers can compare and contrast the different select and sequence choices that their colleagues make. Teachers should work in groups of 2-3.

## Module 4: Comparing and Contrasting Linear, Quadratic, and Exponential Rate of Change

**INSTRUCTOR MATERIAL**



Prior to beginning the task, have teachers look through the suggested “Teacher Moves” and “Sample Responses” (Figure 2) associated with the Avi & Benita’s Repair Shop task, these are shown on the bottom of each screen when looking at the student preview of the task (Figure 3 shows where student preview is in the Dashboard). After glancing through the Teacher Moves, ask: Where do they recommend monitoring students closely? Where do they recommend pausing to have a discussion? And what do they recommend with respect to selecting and sequencing at this point?

The screenshot displays the 'Avi and Benita's Repair Shop' task interface. At the top, there's a navigation bar with tabs for 'Snapshots', 'Summary', 'Teacher', and 'Student'. Below this is a toolbar with icons for 'Anonymize', 'Pacing', and 'Pause'. A row of task steps is visible, with step 7 'Describe...' highlighted. The main area shows a 'STUDENT SCREEN PREVIEW' of a graph titled 'Describe the graphs.' The graph plots 'payment (in dollars)' on the y-axis (ranging from -1000 to 6000) against 'day' on the x-axis (ranging from 0 to 20). Two data series are shown: a red line representing Avi's rule and a blue line representing Benita's rule. The red line starts at (0,0) and increases exponentially, while the blue line starts at (0,0) and increases linearly. To the right of the graph, there are text prompts: 'Describe the graphs.', 'Avi's rule is shown in RED. Benita's rule is shown in BLUE.', 'Now what do you notice about the graphs?', and 'In particular, is there anything you're surprised by?'. Below these prompts is a text input area and a 'Share With Class' button. At the bottom left, there are tabs for 'Teacher Moves' and 'Sample Responses'. A red arrow points from the 'Sample Responses' tab to the graph. A small box at the bottom right contains a note: 'Highlight unique answers for the class, using them as the spark for a brief discussion about the behavior of linear (Benita's rule) vs exponential (Avi's rule) growth. It's up to you whether to introduce the term "exponential" at this stage. You might simply call it "nonlinear" for now.'

Figure 2: Where to find “Teacher Moves”

## Module 4: Comparing and Contrasting Linear, Quadratic, and Exponential Rate of Change

**INSTRUCTOR MATERIAL**



Avi and Benita's Repair Shop [SAMPLE] Inactive Code										Snapshots	Summary	Teacher	Student
Anonymize	Pacing	Pause	1 Repair S...	2 Complet...	3 Extend t...	4 Visualize...	5 Day 20	6 Visualize...	7 Describe...	8 A New Hin...	9 Bonus C...		
17 students	0:4 Time Entered		Avi and Benita run a repair shop.	The table shows the number of cars repaired each day.	Here are the Day 4 answers.	Avi's rule is shown.	Which rule do you think is correct?	Complete the table.	Avi's rule is shown.	Avi and Benita's rule is shown.	Avi and Benita's rule is shown.		
Martina	...	—	•	✓	•	•	✓	•	•	•	•		
Ava	...	—	•	✓	•	•	✓	•	•	•	•		
Alisha	...	—	•	✓	•	•	✓	•	•	•	•		
Jaleel	...	—	•	✓	•	•	✓	•	•	•	•		
Neel	...	—	•	✓	•	•	✗	•	•	•	•		
Anushka	...	—	•	✓	•	•	✓	•	•	•	•		
Emma	...	—	✗	✓	•	•	✓	•	•	•	•		
Javier	...	—	•	✓	•	•	✓	•	•	•	•		
Tyani	...	—	✗	✓	•	•	✓	•	•	•	•		
Maneli	...	—	•	✓	•	•	✓	•	•	•	•		
Sadia	...	—	✗	✗	•	•	✗	•	•	•	•		
Bao	...	—	•	✗	•	•	✗	•	•	•	•		
Mariam	...	—	✗	✗	•	•	✗	•	•	•	•		
Abena	...	—	•	✓	•	•	✓	•	•	•	•		
Kala	...	—											

Figure 3: Where to find “Student Preview” in the Dashboard view

We recommend having teachers do each of Q1-Q2 one at a time discussing choices for each. To facilitate the whole group discussion, have teachers display them publicly by using the present mode in snapshots on their personal laptops (See image below). Then have everyone walk around the room to see what their colleagues selected to allow for comparing/contrasting during the whole class discussion. They can use the “add a question” to add their explanations. If you would like to have their selections and explanations recorded to keep, consider having them paste images in a shared Google Slide or Jamboard. To start the whole class discussion, you might ask teachers to share a snapshot selection that a colleague made that they particularly like, or ask if they noticed any patterns in the group’s selections.

To present selections, press the present button above the snapshots selected (Figure 4).



Present 4 Snapshots
▼

Neel

**Benita's Rule** | ✕

When I sketch the graphs, the blue graph increases faster but the red

Neel

Emma

**Avi's Rule** | ✕

At first I thought that Avi's went up by 1 cent each day, but on number 3 I

Emma

Figure 4: Where to find “Present Snapshots”



## 4.3 Sample Responses

### Selecting and Sequencing: Avi & Benita's Repair Shop

**Q1.** Paste an image of the snapshots of students' responses you selected and sequenced for a class discussion for **page 5** of the task. Explain your select and sequence strategy.

Ideal answers should include graphs & text responses with clear explanations on why the teacher selected and sequenced in the order presented. Responses will vary:

## Slide 5:

5 Sadia X      5 Abena X

5 Javier X

I chose this order of students because the first and second both draw a linear functions to represent the growth in pay. I would ask students to explain how they calculated day seveeteens pay for Benti and then Avi. I chose the last graph because Javier was able to understood the correct calculations and graph.

Avi starts paying so little amount initially. So, I don't think it will pass Benita's amount on the 20th day although Avi doubles the amount every day.

From the previous table and the graphs, I can see that Benita pays more and more. Avi pays more but still very small increase

Avi's rule increases exponentially and doubles in amount each day. By day 20, she will pay \$5,242.88

Benita's rule increases in a linear fashion and increases only by 100 each day. By day 20, she will pay \$2,000

I chose these three students statements to have a discussion with. I chose student one because they recognize that Avi's rule doubles but don't completely understand that his rule will surpass that Benita's after day 18. I chose the next statement because this students wording is a little misunderstanding s I would ask her to explain her reasoning to see why she states "Avi pays more but still very small increase." I chose the third students statement because they use math to explain their reasoning. They understand that Avi's rule is increasing expontially and will surpass Benita's rule.

### 1<sup>st</sup> Sample Teacher Response to Q1





**Slide  
5**

**Collection 1**

**Benita's Rule**

Benita gets more each day

**Benita's Rule**

When I sketch the graphs, the blue graph increases faster but the red graph doesn't increase as fast

**Avi's Rule**

At first I thought that Avi's went up by 1 cent each day, but on number 3 I figured out that his is doubling. Benita's looks like it is going up faster, but if you guess the points, it looks like Avi's will go up faster later.

**Collection 2**

Three graphs showing payment (in dollars) vs day (0 to 20). The top two graphs show Benita's payment (blue line) increasing faster than Avi's (red line). The bottom graph shows Benita's payment (blue line) increasing faster than Avi's (red line).

I chose this order because I want students to first challenge responses that may not be fully thought out yet, and then I want the students to see a way of thinking that leads to the correct answer.

I want students to identify the issue with the first graph. It's not a common answer. Then, I want to explore the idea of Avi's model being linear. Finally, we will discuss how Avi's model is non-linear.

2<sup>nd</sup> Sample Teacher Response to Q1





**Q2.** Paste an image of the snapshots of students' responses you selected and sequenced for a class discussion for **page 7** of the task. Explain your select and sequence strategy.

Responses will vary:

## Slide 7:

I was surprised by the red graph increased faster than the blue on day 20 because there was no increase in the red until day 10 and then it got ahead of the blue!!

Avis grew real fast after day 16. I never thought that would happen

I added 100 for each day in Benita's plan and it ended at 2000 but started multiplying by 2 Avi's payment and it stopped at 524.288 for day 20 it was not surprising how awful is Avi's payment

I chose these three statements as my discussion pieces. I chose the first one because the student notices that the Avi's rule increases faster but not until after a certain day. I chose the second statement because the student explicitly states "I never thought would happen" I would address his statement and ask him why he might have thought this. I chose this last statement because this student is doing the correct operation of multiplying by two but isn't getting the correct calculations for Avi's rule.

1<sup>st</sup> Sample Teacher Response to Q2



I didn't think Avi's would ever get passed Benitas

Benita's is still going up the same each time. Avi's makes big jumps after day 15 and passes Benita after day 18. I am surprised at how one cent turns into over \$5000 in less than three weeks.

Avi's rule increases exponentially so it started its increase more gradually than Benita's rule, but overall grew faster

#7 I picked the order above because the first student didn't think Avi's would ever pass Benita and my question would be why did they think that. Then student two relates their findings to the graph and mentions when Avi's passes Benita's and they're shocked by their findings. The last student recognized that Avi's is exponential so it will gradually pass Benita's and grow quickly at some point.

2<sup>nd</sup> Sample Teacher Response to Q2