



## Qualitative Analysis of Representations of Functional Relationships

**Total time:** 150 minutes but could be shortened to 75 minutes if needed. See page 2 for recommendations for a 75-minute module.

**Situating the Module within the PTMT Algebra Materials:** This module could be used alongside Chapter 5: Rate of Change of the PTMT Algebra Materials. Section 1: Experiencing Rates of Change is focused on leveraging teachers' experiences with movement and building simulations of movement to discuss and represent rates of change. If using both sets of materials, we recommend completing this module first and then moving to Section 1 of the Algebra materials where teachers will consider another simulation (i.e., a series of interesting elevators) and learn about how to use motion detectors to collect data and create graphical representations of their own movement.

**Overview of the Module:** This module is designed around a similar task that Desmos has built called Function Carnival. The Function Carnival task includes three simulations: Cannon Man, Cars, and Ferris Wheel, which engage students in analyzing the movement of each ride and representing the corresponding features of rate of change and functional relationships. In this module teachers will first engage with Function Carnival as learners. Teachers will discuss affordances of the task, share their experiences, and discuss which student's thinking they found to be the most difficult to interpret. Teachers will then anticipate the first graph students may draw for each of the three simulations; this will be managed by the instructor with a Desmos task. After each anticipation, teachers will discuss their anticipations while the instructor models how to use the Desmos Teacher Dashboard. For homework, teachers will complete an assignment that asks them to watch a series of videos of different teachers engaging with the simulations and Notice and Wonder about the teachers' thinking. On the second day, teachers will focus on the sequence of one pair of students' graphs as they work through the Cars simulation. Finally, teachers will work in pairs to complete a task that has them mimic using the Desmos Teacher Dashboard to practice the monitoring phase of the 5 Practices with different student pairs' graphs from the Cars simulation.

### Module Goals:

- Build an understanding of the ways in which relationships between changing quantities define functions and the ways in which we can represent such relationships graphically. Specifically, making sense between graphical representations and the underlying mathematical ideas related to changing quantities (i.e., quantities changing together).
- Examine student practices when working within the Function Carnival Desmos Task.
- Consider how to use the Desmos Teacher Dashboard to pace your students on a technological task.
- Practice the pedagogical skills of anticipating, monitoring, and selecting and sequencing students' work (Smith & Stein, 2018) within the Desmos Teacher Dashboard.



Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 0	Homework	3.1 Function Carnival Task
	Day 1	15 min	3.1 Discussion
		60 min	3.2 Anticipation of Student Work and Introduction to Desmos Teacher Dashboard
			3.2a Anticipate Cannon Man
			3.2b Anticipate Cars
			3.2c Anticipate Ferris Wheel
		Homework	3.3 Noticing Student Thinking: Function Carnival Task
	Day 2	30 min	3.4 Analyzing & Supporting Emergent Student Thinking (Only the Cars Simulation of Function Carnival)
		45 min	3.5 Monitoring Students' Work on the Function Carnival Task (Only the Cars Simulation of Function Carnival) Either needs to be homework or completed as a whole class through discussion and watching the videos.

**Recommendations for 75-minute module:**

This module is designed for 150 minutes but could easily be shortened to 75 minutes if needed. Suggestions include:

- One approach is to engage teachers in anticipating only the Cars simulation (instead of all three: Cars, Cannon Man and Ferris Wheel) because both the Analyzing and Supporting Emergent Student Thinking (3.4) and the Monitoring Students' Work (3.5) tasks are based on Cars.
- Tasks 3.3–3.5 are not sequential. Therefore, you can choose to change the order or remove any of these assignments to fit your course. If you choose only one task, we recommend 3.5 Monitoring Students' Work on Function Carnival since it provides an approximation of the practices of monitoring, selecting and sequencing students' work.

**MTE preparation for Module 3:**

- Engage with the Desmos 3.1 Function Carnival Task.
- (Re) Familiarize yourself with the 5 Practices for Orchestrating Productive Discussions.
  - Suggested Refresher: Nabb, K., Hofacker, E. B., Ernie, K. T., & Ahrendt, S. (2018). Using the 5 practices in mathematics teaching: Selecting and sequencing student work with cognitively demanding tasks in a group environment can teach important mathematical ideas. *Mathematics Teacher*, 111(5), 367–373.
  - For Digging Deeper: Smith, M., Steele, M. D., & Sherin, M. G. (2020). *The 5 practices in practice: Successfully orchestrating mathematical discussions in your high school classroom*. NCTM. Chapters 3–5.





- Read commentaries, engage with assignments (including watching associated videos), and look over sample responses.
- For a Deeper Dive into the Mathematical Ideas: Johnson, H. L., McClintock, E. D., & Gardner, A. (2020). Opportunities for reasoning: Digital task design to promote students' conceptions of graphs as representing relationships between quantities. *Digital Experiences in Mathematics Education*, 6, 340–366. <https://doi.org/10.1007/s40751-020-00061-9>








### Suggested Readings for Teachers:

- Johnson, H. L, Hornbein, P., & Azeem, S. (2016). Investigating functions with a Ferris Wheel. *Mathematics Teacher*, 110(5), 345–351.
- Nabb, K., Hofacker, E. B., Ernie, K. T., & Ahrendt, S. (2018). Using the 5 practices in mathematics teaching: Selecting and sequencing student work with cognitively demanding tasks in a group environment can teach important mathematical ideas. *Mathematics Teacher*, 111(5), 367–373.

### Detailed Agenda for Module 3

	Description of Module Tasks	Facilitation Notes
Day 0 / Homework	<b>3.1 Function Carnival Task</b>   PSMT devices  <a href="#">Desmos Task</a>  Teachers complete the Desmos task that has them engage with the three simulations (Cars, Cannon Man and Ferris Wheel) and create graphs representing them.	Create a class code for the <a href="#">Desmos Task</a> and provide the link to your students.  NOTE: Desmos Teacher Dashboard is different for this file since it was created in an older version of Desmos.  We recommend assigning Johnson, Hornbein, & Azeem (2016) as a reading for AFTER teachers complete the task as a learner. This practitioner article focuses on the mathematics in a similar Ferris Wheel task and will help teachers make sense of the overarching mathematical goals in this module.  See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.





Day 1	<p><b>3.1 Discussion</b> (15 minutes)</p> <p> PSMT devices   <a href="#">Desmos Task</a></p> <p>Teachers participate in a whole class discussion about the different carnival tasks (e.g., which they found most simple, most challenging)</p>	<p>Have the teachers login to their Desmos account and review their work on the different carnival tasks. We suggest allowing small groups of teachers to discuss prior to opening the discussion to the whole class. Have the teachers discuss the task they found most difficult for themselves as a learner and then which students' thinking they found to be the most difficult to interpret.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>
	<p><b>3.2 Anticipation of Student Work and Introduction to Desmos Teacher Dashboard</b> (60 minutes)</p> <p><b>3.2a Anticipate Cannon Man</b>  <b>3.2b Anticipate Cars</b>  <b>3.2c Anticipate Ferris Wheel</b></p> <p> PSMT devices   <a href="#">Desmos Task</a></p> <p>Individually:   <a href="#">Desmos Task 3.2a</a>   <a href="#">Desmos Task 3.2b</a>   <a href="#">Desmos Task 3.2c</a></p> <p>Teachers anticipate the first graph students may draw for each of the three simulations. As teachers engage in their anticipations, their responses are managed by the instructor using a Desmos Activity Builder file which is used to situate whole class discussions.</p>	<p>Create a class code for this <a href="#">Desmos Task</a> and provide the link to your students. We recommend asking teachers to log in so that the task will appear in their history, and they can revisit it at any time. In addition, they must be logged in to receive any feedback you provide.</p> <p>This task was created using Desmos Activity Builder, so instructors could use the opportunity to introduce teachers to the Teacher Dashboard during a whole class discussion of their own monitoring while the teachers are working.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>



Day 1 / Homework	<p><b>3.3 Noticing Student Thinking: Function Carnival Task</b></p> <p>📱 PSMT devices</p> <p>🔗 <a href="#">Worksheet</a></p> <p>For this homework assignment, teachers watch videos of pairs of students engaging with the Cannon Man and Cars tasks. Teachers note something smart about each student's graph, and then attend to and interpret each pair's understanding of rate of change.</p>	<p>Provide a copy of the electronic worksheet to the teachers. We recommend pointing teachers to the NITE framework and reminding teachers that when noticing student thinking with technology, they should pay careful attention to what students say, do with the technology, and what they record.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>
Day 2	<p><b>3.4 Analyzing and Supporting Emergent Student Thinking</b> (30 minutes)</p> <p>📱 PSMT devices</p> <p>🔗 <a href="#">Worksheet</a></p> <p>Teachers watch a sequence of videos of one pair of students as they work through the Cars simulation. Teachers reflect on a) what the students do with the technology, b) what they see with the technology, and c) say to each other as they work to make sense of creating a graphical representation of the car simulation. Then, teachers are asked what questions they might pose to assess the students' understanding and support them in making connections between their graph and the underlying mathematical ideas.</p>	<p>Provide a copy of the electronic worksheet to the teachers. We recommend that teachers first work in small groups on the task. Then facilitate a whole class discussion to bring out the big ideas with a focus on the last question, because while the students in the video did figure out how to create a correct graph, it is not clear that they understand the mathematical relationship between the car's movement and its representation on the graph. Focus the discussion on teacher moves that would help both assess and advance these students' thinking related to the mathematical goal.</p> <p>Note: If teachers have not had a lot of practice with noticing students' thinking in a technology-mediated learning environment, it may be helpful to provide more scaffolding for the attend and interpret questions by separating spoken and written, and technology engagement into two different questions, so teachers focus on addressing all elements.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>



<b>Day 2 (cont.)</b>	<p><b>3.5 Monitoring Students' Work on the Function Carnival</b> (45 minutes)</p> <p> PSMT devices</p> <p> <a href="#">Worksheet</a></p> <p>Teachers practice monitoring, selecting and sequencing 6 student groups' work within the Desmos Teacher Dashboard. They list their first impressions of each graph created by the students, and then watch videos of each group engaged with the task. After watching the videos, teachers are reminded of the task's learning goals and then asked to select and sequence three groups that they would have present.</p>	<p>Provide a copy of the electronic worksheet to the teachers We recommend teachers complete this task in pairs or small groups. After the teachers finish their selection and sequencing, we recommend having a whole class discussion focused on how the different selections and sequences were chosen to build toward the learning goals.</p> <p>See the full <a href="#">Instructor Materials</a> which include the facilitation notes and sample responses.</p>
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