Module 2 Overview Document

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 1	May Vary	Optional Pre-Work with Algebra Tiles
		45 min	2.1 Completing the Square with Algebra Tiles
		20 min	2.2 Completing the Square with CAS
		10 min	Compare and Contrast 2.1 and 2.2
		Homework	2.3 Noticing Student Thinking: Completing the Square with Virtual Algebra Tiles
	Day 2	15 min	2.3 Discussion
		20 min	2.4 Posing Purposeful Questions: Completing the Square with Virtual Algebra Tiles
		30 min	2.5 Noticing Student Thinking: Completing the Square with CAS
		10 min	Revisit Compare and Contrast 2.1 and 2.2
		Homework	Read the following two articles: Lischka, A. E., & Stephens, D. C. (2020). The area model: Building mathematical connections. *Mathematics Teacher: Learning and Teaching PK-12*, *113*(3), 186-195. Zbiek & Heid (2009) Using Computer Algebra Systems to Develop Big Ideas in Mathematics. *Mathematics Teacher*, *(102)*7, 540-544.

2.4 Facilitation Notes

The task document (2.4 Posing Purposeful Questions: Completing the Square Using Virtual Algebra Tiles) should be shared electronically with teachers as it contains hyperlinks to videos

This task provides teachers with the opportunity to think carefully about posing purposeful assessing and advancing questions to pairs of students who have used the technology differently to model the same expression.

Note: The included student work represents the variety of strategies we saw with integrated Math 3 students being introduced to completing the square.

If teachers are not familiar with posing purposeful questions and the differences between assessing and advancing questions, we recommend reading NCTM's *Taking Action Grades 6-8* or *Taking Action 9-12 Chapter 5: Pose Purposeful Questions*.

We recommend teachers work in small groups to complete Q1-Q4. Prior to answering Q5, we recommend stopping for a whole-class discussion making sure to highlight connections between the models and expressions each pair of students produced as well as the various ways in which algebra tiles were used to model the expression.

Have teachers share how their proposed assessing and advancing questions differed based on the students' various models.

2.4 Sample Responses
Posing Purposeful Questions: Completing the Square with Virtual Algebra Tiles

In this task you will examine students' work on the Completing the Square with Virtual Algebra Tiles task.

The students have all had prior experience modeling expressions using algebra tiles. This includes using algebra tiles to model factoring quadratic expressions. The learning goal for the lesson that includes this task is that students will understand what it means to complete the square for a quadratic expression and the performance goals include:

- The students will be able to build area models of quadratic expressions using algebra tiles to complete the square.
- Students will be able to write an equivalent expression with a perfect square (i.e., complete the square) when given a quadratic expression.
- Students will be able to explain why the area models of the original quadratic expression and the completed square expression are equivalent.

Introduction to Completing the Square with Algebra Tiles

The images that follow are the models created by four pairs of Math 3 students. For each response, based on the models, list at least one assessing and one advancing question you would like to pose to further assess their current understanding. Explain your reasoning.

Q1. Emilee and Katelyn's model for $x^2 + 12x - 7$ is shown below (Figure 1). List one assessing and one advancing question you would like to ask Emillee and Katelyn to further understand their thinking. Explain.

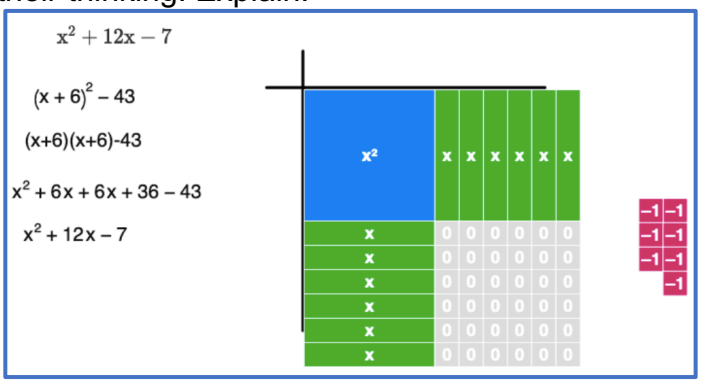

Figure 1: Emilee and Katelyn's model for $x^2 + 12x - 7$

Sample Assessing Questions:

- How did you determine the number of negatives you would need? Rationale: Evidently, their solution here is correct. However, the question will help to determine if this was a result of understanding the core idea behind it, instead of pure luck.
- Where did 43 in your equation come from? This will help students to explain where all of their negative 1 tiles are at on the picture.
- How do your algebra tiles model the written parts of your work? I want to assess their understanding of how the algebra tiles relate to the work they have written.
- What role does a negative constant term (c) play in the creation of completing the square? I would ask this question to see their understanding of how that negative would affect the equation and what they think would happen in different scenarios.
- How does b in bx affect c and what the ending equation would look like? Trying to see if they made the connection between the equation itself and the process of how to complete the square for other expressions.

Sample Advancing Questions:

- Can you solve it in a different way? Rationale: This will force them to think differently, to build a different perspective of the problem.
- What if the -7 was a $+7$? They had to think a bit about how to handle adding in so many 0s, it would be a good opportunity to think about that some more and get more comfortable with it.
- I think I'd give them an example similar to this one so they had a chance to think about how to handle a situation that needs so many 0s added in again. Something like $x^2 + 18x - 2$. Having such a big "b" might help them start to think about working without the tiles but imagining the square.

Q2. Kei and Xarielle's model is shown below (Figure 2). List one assessing and one advancing question you would like to ask Kei and Xarielle to further understand their thinking. Explain.

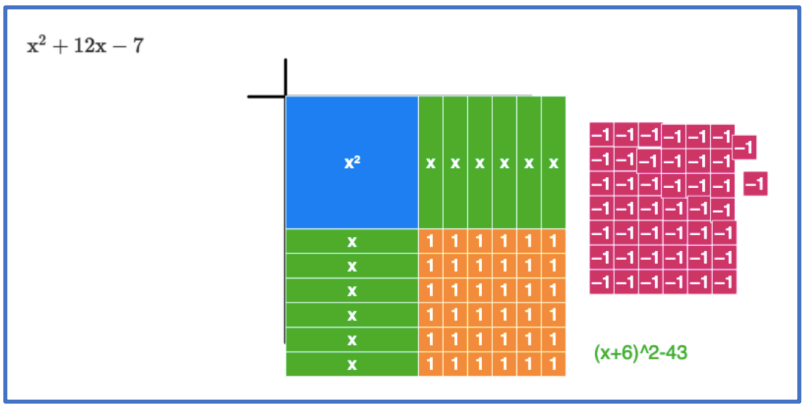

Figure 2: Kei and Xarielle's model for $x^2 + 12x - 7$

Sample Assessing Questions:

- What role do the negative ones play in the solution? Rationale: To see if the students are able to explain that a positive one and negative one forms a zero, so they had to use enough negative ones to offset filling the corner of the square with positive ones with the seven remaining negative ones coming from the original expression.
- How did you use the algebra tiles to find your answer? I want to know if they understand the connection of the model to the expression and catch the difference in their algebra tiles and their answer.
- How do you know your answer is equivalent to $x^2 + 12x - 7$? I want to see if they understand that it is because when expanding $(x + 6)^2 - 43$ they get the original expression.

Sample Advancing Questions:

- Is there another way you could use the algebra tiles to model the expression? I am hoping they will combine the negative ones and positive ones together to organize their picture in a second way to model the expression.
- When we combine the negative ones and positive ones, what is leftover? Does this match the expression at the beginning? I could ask this and walk away to give them time to see if it matches their original expression, and they should see they have an extra negative one tile.

Q3. Amelia and Isaiah's model is shown below (Figure 3). List one assessing and one advancing question you would like to ask Amelia and Isaiah to further understand their thinking. Explain.

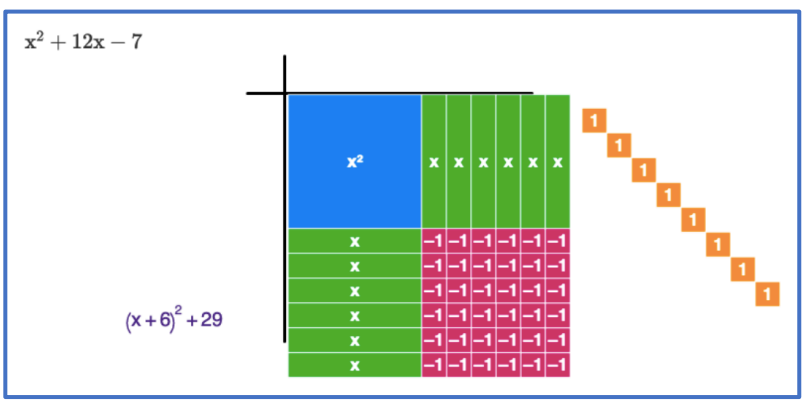

Figure 3: Amelia and Isaiah's model for $x^2 + 12x - 7$

Sample Assessing Questions:

- How did you use the algebra tiles to model the original expression then come up with your answer? I want to understand more about why they chose to use negative one tiles to fill in the square and put the eight

positive one tiles to the side. Honestly, I just need to know more about what they are thinking because $-36 + 8$ is not even 29.

- What made you decide to fill in the corner of your square with negative one tiles? This will help me understand what they were thinking since you cannot multiply two positive numbers and get a negative.
- How did you come up with $+29$ in your answer? Rationale: To have them talk me through their work with the algebra tiles and hopefully realize they need to think about their work more.

Sample Advancing Questions:

- What would be the result when you expand your answer? Does it match the original expression? The idea is to have them realize that, since they are not equal, they need to think more about their work and revise.
- Choose a value for x and substitute it into both the original expression and your solution for completing the square. Are the output values the same? Should they be the same? Rationale: To help them see that the value for the expressions must be the same to be equivalent, but they are not, so they need to think more about their work.

Q4. Addison and Allison's model is shown below (Figure 4). List one assessing and one advancing question you would like to ask Addison and Allison to further understand their thinking. Explain.

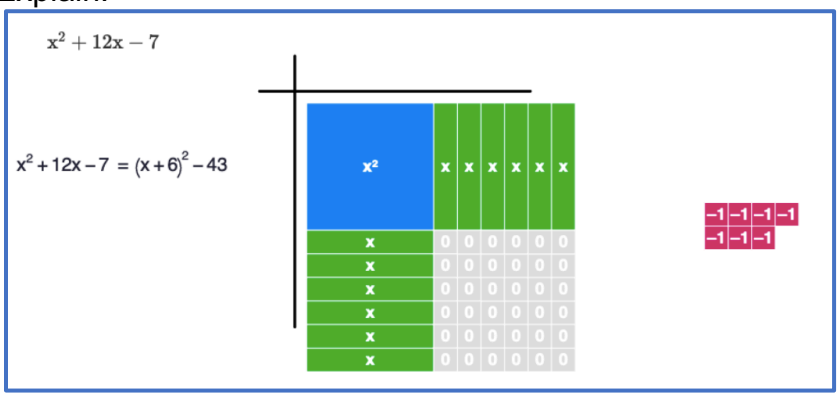

Figure 4: Addison and Allison's model for $x^2 + 12x - 7$

Sample Assessing Questions:

- How did you use the algebra tiles to find your answer? I want to know if they understand the connection of the model to the expression. This is a way to see if they show deep conceptual understanding.
- What made you decide to put the negative ones on the positive ones? This would allow them to explain how many they have of each and how they would sum to zero within the expression they give on the side so they want to reflect that in their model.
- How do you know both expressions are equivalent? Rationale: To test their understanding of the underlying mathematics.

Sample Advancing Questions:

- Is there another way you could use the algebra tiles to model the expression? I am hoping they will arrange the positive ones in the square and leave all the negative ones to the side, maybe as a block of -36 and the -7 separately.
- What would happen if the original expression was $x^2 - 12x - 7$? Rationale: This question will push them to think about what they would do if the coefficient of x was negative.

Q5. NCTM describes four different types of question foci (p. 76 in the Pose Purposeful Questions chapter you read for homework).

- Gathering information questions ask students to recall facts, definitions, or procedures.
- Probing thinking questions ask students to explain, elaborate, or clarify their thinking.
- Making mathematics visible questions ask students to discuss mathematical structures and make connections among ideas and relationships.
- Encouraging reflection and justification questions ask students to reveal understanding of their reasoning and actions or make an argument for the validity of their work.

Look back over the assessing and advancing questions you listed for Q1 through Q4 and identify the foci of each question you asked. If you did not ask any of a particular foci, go back and write a question for a student groups of choice.

Responses for each type should either include questions posed in Q1 - Q4, or may be a new question if they did not write one or more of the types of questions. Some examples follow:

- Gathering information questions ask students to recall facts, definitions, or procedures.
  - What happens when you combine a negative one tile and a positive one tile?
- Probing thinking questions ask students to explain, elaborate, or clarify their thinking.
  - Where did 43 in your equation come from?
- Making mathematics visible questions ask students to discuss mathematical structures and make connections among ideas and relationships.
  - How do your algebra tiles model the written parts of your work?
- Encouraging reflection and justification questions ask students to reveal understanding of their reasoning and actions or make an argument for the validity of their work.
  - How do you know both expressions are equivalent?