



## 2.2 Completing the Square with CAS

### Investigating Equivalent Expressions

In this investigation you are going to use a Computer Algebra System (CAS) tool. CAS tools can do symbolic manipulations. The two commands you will use today are “factor” and “expand”.



[Computer Algebra System \(CAS\)](#)

	Factor( $x^2 - 4$ ) $\rightarrow (x - 2)(x + 2)$
	Expand( $(x + 2)^3$ ) $\rightarrow x^3 + 6x^2 + 12x + 8$
	Input...

For example, typing **factor**( $x^2 - 4$ ) into a CAS tool will result in  $(x - 2)(x + 2)$ . Similarly, entering **expand**( $(x + 2)^3$ ) will result in  $x^3 + 6x^2 + 12x + 8$ . Importantly, you must put the entire expression you would like expanded or factored in parentheses.

Before moving on, try it!

**Q1.** Complete the table below by determining the missing equivalent expanded or factored form of each expression. You may use the CAS tool if you'd like.

Table 1: Complete the table with missing equivalent expressions

Factored Form	Expanded Form
$(x + 1)(x + 1)$	
$(x + 2)(x + 2)$	
$(x + 3)(x + 3)$	
	$x^2 + 6x + 9$
	$x^2 + 12x + 36$
	$x^2 + 14x + 49$
	$x^2 + 24x + 144$



**Q2.** Look at your completed table. What do you notice? What do you wonder? Please write down at least 3 things that you notice.

A quadratic expression whose linear factors are the same is called a perfect square. For example,  $x^2 + 2x + 1$  is a perfect square because its factors are  $(x + 1)(x + 1)$  or  $(x + 1)^2$ , and as is shown below (Figure 2), it actually forms a square (i.e., the length and width are the same)!

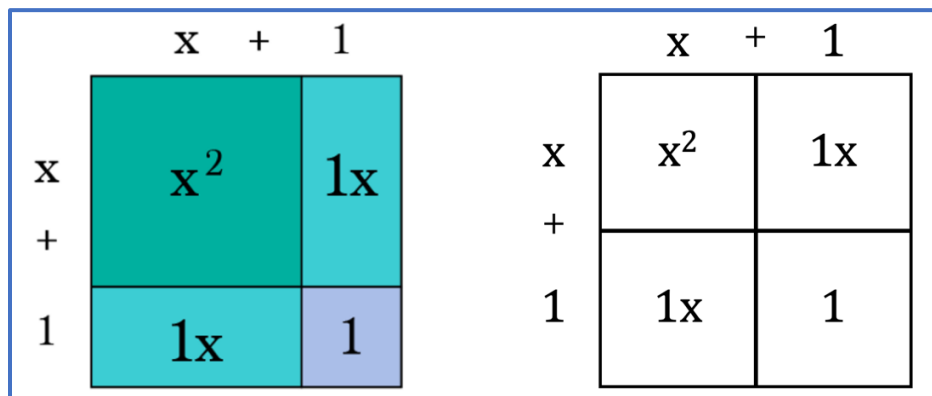


Figure 2: Model of the Perfect Square  $(x + 1)^2$

**Q3.** Using the patterns you noticed above, predict the missing terms for each equation below to make the two expressions equivalent by completing the square. Check each using your CAS.

Table 2: Complete the table by predicting the missing terms

$x^2 + 16x + \underline{\hspace{1cm}}$	$= (x + 8)^2$
$x^2 + \underline{\hspace{1cm}} + 81$	$= (x + 9)^2$
$x^2 + 4x + \underline{\hspace{1cm}}$	$= (x + \underline{\hspace{1cm}})^2$



$$x^2 + \underline{\hspace{1cm}} + 100 = (x + \underline{\hspace{1cm}})^2$$

$$x^2 + 22x + \underline{\hspace{1cm}} = (x + \underline{\hspace{1cm}})^2$$

**Q4.** Explain to a friend how to complete the square for the expression  $x^2 + bx + \underline{\hspace{1cm}}$ . Make sure you explain both how to complete the square and how to determine the equivalent squared expression.

**Challenge!** 😊

What if the expression you are trying to complete the square for is in the form  $x^2 - bx$ ?

Use the CAS tool to explore with some examples, and then based on your observations explain to a friend how to complete the square and how to determine the equivalent squared expression.