



Module 1 Overview Document

Table 1: Timeline of Tasks in the Module

Timeline of tasks in the Module	Day 1	75 Minutes	1.1 Comparing and Contrasting Tasks and Technology with a Focus on Digital Equity Read: McCulloch, A. W., Lovett, J. N., Dick, L. K., & Cayton, C. (2021). Positioning each and every student as a mathematical explorer with technology. <i>Mathematics Teacher: Learning and Teaching PK-12: Special Issue on Digital Equity and the Digital Divide</i> , 114(10), 738–749.
		Homework	1.2 Noticing Student Thinking: Quadratic Functions - Making Sense of “h”
	Day 2	30 Minutes	1.3 Introduction to Noticing Students’ Mathematical Thinking in a Technology-Mediated Learning Environment
		Homework	Read: Dick, L. K., McCulloch, A. W., & Lovett, J. N. (2021). When students use technology tools, what are you noticing? <i>Mathematics Teacher: Learning and Teaching PK-12</i> , 114(4), 272–283.

1.3 Facilitation Notes

Start the discussion by noting one of the most important things you do as a teacher is notice and build on students’ mathematical thinking during instruction. Noticing student thinking is more complex when technology is involved.

Next, we recommend creating a list of what teachers ‘noticed’ in their responses to #1 and #2 on task 1.2.



Then, use the [PowerPoint for introduction to framework](#) to guide the rest of the discussion.



1.3 Commentary

Introduction to Noticing Students' Mathematical Thinking in a Technology-Mediated Learning Environment

Before formally introducing teachers to the construct of noticing student's mathematical thinking in a technology-mediated learning environment, have them complete the 1.2 Noticing Student Thinking - Quadratic Functions: Making Sense of "h" task (this is ideally completed as homework). They will watch a brief video of a pair of students, Sara and Julian, working on a task in which they are being introduced to parameters of quadratic functions in vertex form.



Once teachers have completed this task, you can use the [Introduction to Noticing PowerPoint](#) to guide your discussion (slide descriptions are shown below).

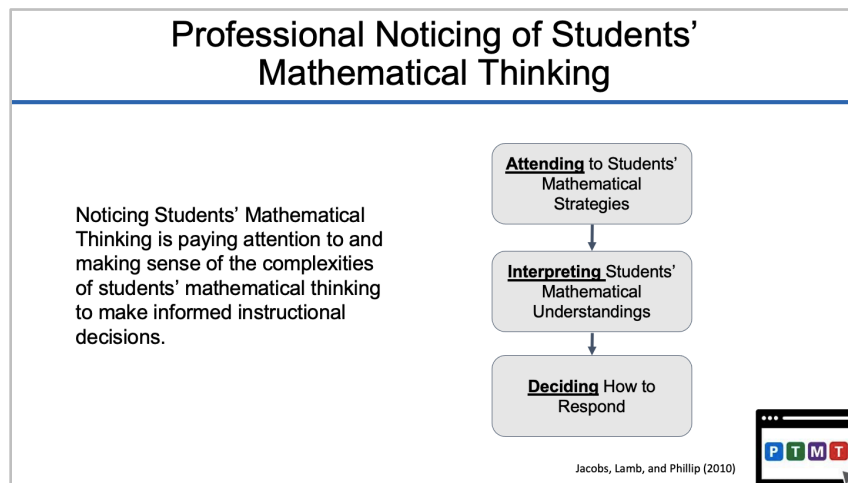
Table 2: Notes to guide your discussion when using the slides

Slide	Notes
1	Title Slide
2	<p>The intent of this slide is to use the NCTM Mathematics Teaching Practices, specifically eliciting and using student thinking, as grounding for the importance of learning to notice. Noticing is essentially going through this process, attending to students' mathematical strategies, interpreting their understandings, and then using that information to adjust your instruction.</p> <div data-bbox="284 1197 1161 1690"> <p style="text-align: center;">Mathematics Teaching Practices</p> <p style="text-align: right; font-size: small;">NCTM's Principles to Actions (2014)</p> </div>



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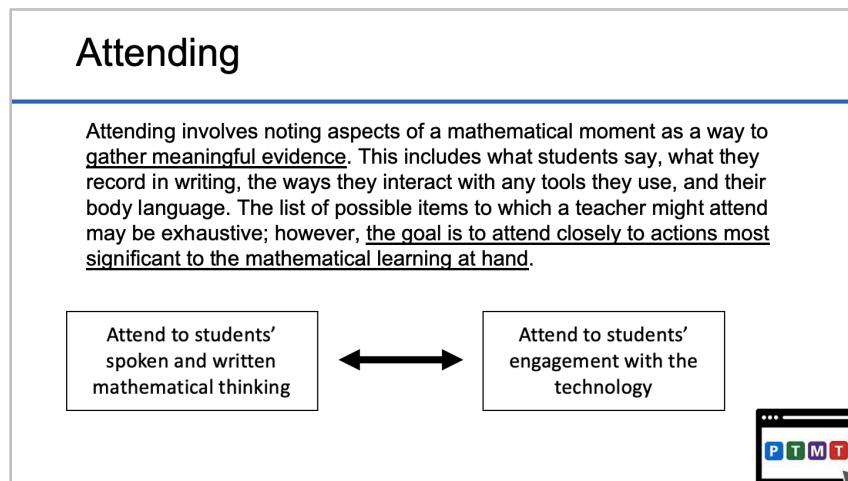
Here we introduce the 3 components of Professionally Noticing of Students' Mathematical Thinking as defined by Jacobs et al. (2010). The three components are intertwined, as they inform each other. We will refer to the practice generally as "noticing" and the three components as "Attend", "Interpret", and "Decide how to respond".



4

Attend is often thought of as a play-by-play of what the student does while making sense of the mathematics task. (We suggest using that phrase as a reminder for teachers to include detail, not just broad generalizations)

We have separated attending to students' written and spoken work and their engagement with the technology to highlight that teachers need to consider both and make sense of the coordination of the two when they are attending. For the PTMT-ESP modules, attending to students' thinking always involves their engagement with technology, thus this coordination is needed whenever the teachers are noticing.



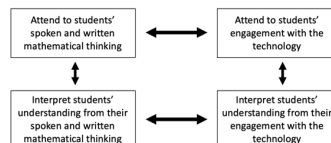


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Similar to attending, for the PTMT-ESP modules, interpreting students' thinking always involves their engagement with technology. A major component of evidence is how the students' engage with the technology. Thus, coordinating the students' written and spoken work, and their technological engagement is needed when noticing.

Interpreting

Interpreting involves coordinating the observed actions (from attending) with what is known about mathematical development in a particular area. The key to meaningful interpretation is making a strong connection to the evidence gathered while attending.



Note: A teacher is not expected to construct a complete picture of a student's understanding based on a single problem (or excerpt), but rather one that is consistent with both the details of the specific student's strategies in a particular moment.



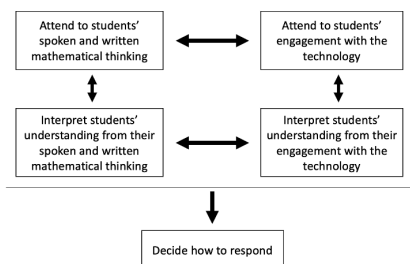
6

We have separated ***decide how to respond*** from the other components of noticing to balance the importance of focusing on both spoken and written mathematical thinking, and tool-engagement prior to making instructional decisions; if a teacher focuses on one more than the other, then they may not be fully informed when making an instructional decision. In addition, when deciding how to respond to a student working in a tool-mediated environment, the teacher must consider how to position the tool (or not) in their response to support the student in moving forward. For this reason, deciding how to respond does not necessarily include students' engagement with a tool.

Deciding How to Respond

Deciding how to respond refers to conceiving (and executing) pedagogical decisions drawn from the interpretation of a student's mathematical thinking.

Decisions should be directly connected to your interpretations and your mathematical goals.

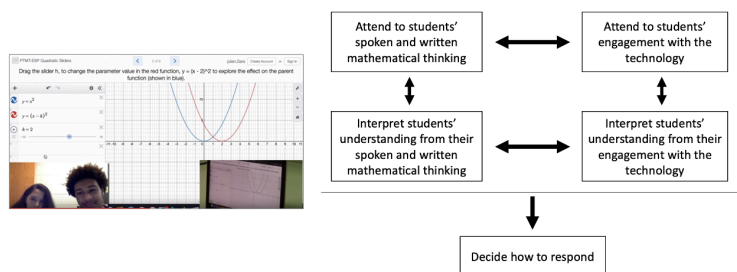




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Here ask teachers look back at their **attend** and **interpret** responses from earlier. How do they think they did? What would they notice differently if they were to watch the video again?

Noticing student thinking in a technology-mediated environment



Dick, Lovett, & McCulloch (2021)



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Transition Slide

In the following slide teachers are going to compare and contrast two different **attend** statements - the second one being guided by the framework. It might be helpful to watch the video again, as a reminder, prior to this comparison.

9

Ask teachers to compare/contrast the two attend statements. Record what they note on the board.

If it does not come out in their comparison, point out that in example 2 you can clearly envision the students as they work on the task (i.e., a play by play). The important mathematical “moves” they made verbally, in writing, and with the technology tool are described.

Example 1:

Sara and Julian drag the h slider and decide it moves the graph left and right, but they graph it going the wrong direction.

Example 2:

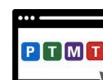
As they clarify what the goal is with the teacher, Sara is dragging the h slider from -10 to 10 and they appear to be following the movement of the red graph as it moves first left of the origin and then right of the origin. Julian says, “it changes the value of it”, but it is not clear what “it” is. Then the teacher asks “how does it change the value of it”? Dragging the slider from 0 to 1, Sara then says “it changes where the x is” referring to the x value of the vertex of the parabola. Julian agrees and explains, “where it first was, was at the zero which is the origin.”

Next, Sara moves the slider from 1 to 2 and says, “it would be x minus 2 to the second” indicating she sees the expression at the top of the page that is changing dynamically as she drags the slider. Then Julian explains H is the “2” in “ $x - 2$ ”.

Julian (pointing at vertex of the red graph on the screen) says “see how if this is 2 and you subtract the 2 you would still have the zero.” Sara asks Julian “what about the raised to the 2nd power”? He explains you first have to do what is in the parentheses and you are subtracting and going back to the origin. Then if you square zero you get zero. He then adds that if it was “in there with the h ” that would be different.

We then see them trying to graph $y = -(x-2)^2 + 3$. Sara says H is the two, Julian begins to place a point at (2, 3) and then changes his mind and begins to move his hand to the left of the origin to graph the vertex. Sara stops him and suggest they go back to look at the slider again. They quickly drag the h slider and note it moves the graph of the function left and right. They go back and graph the point (-2, 3) for the vertex of the parabola.

Compare and Contrast: Attend Statements





10	<p>Transition Slide</p> <p>In the following slide teachers are going to compare and contrast two different interpret statements—the second one being guided by the framework.</p>
11	<p>Ask teachers to compare/contrast the two interpret statements. Record what they note on the board.</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p>Example 1:</p> <p>Sara and Julian understand that h shifts the graph of the function left and right. They think if h is negative the graph moves left. This is evident from the way they graph the vertex at the end of the video.</p> </div> <div style="width: 45%;"> <p>Example 2:</p> <p>Sara and Julian understand that h shifts the graph of the function left and right, and does not affect the vertical shift. This is evident in the way they both test and explain the relationship between the value of the slider, the position of the vertex, and the form of the function. For example, when h is 2 they note the value of h is subtracted from x and the result is squared which explains why the vertex is still on the x axis when they change its value.</p> <p>While there is evidence that they understand the relationship between the value of h in the function and the location of the vertex, this understanding is not robust as they did not apply it accurately when they went to graph the example. They seem to have forgotten that h is actually $x - h$ and instead think of it as $x + h$. This may be because when they went back to double check they just quickly dragged the slider, noting the graph moves left and right and did not look at the function at the top of the page again.</p> </div> </div> <p style="text-align: center;">Compare and Contrast: Interpret Statements</p> </div>
12	<p>Emphasize that the “decide how to respond” component does not need to include the use of technology, and decisions should be directly connected to their attend and interpret components.</p> <p>Ask for other possible ways to respond to the students. Remind them that responding to students does not have to include using the technology and responding is NOT telling students right/wrong, but rather making a strategic pedagogical move. Note: We will spend a lot of time practicing attending and interpreting because it is a very important precursor to making instructional decisions based on students’ thinking.</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <h3 style="text-align: center;">Decide How to Respond</h3> <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p>Since our interpretation is drawn from both what Julian and Sara said and what they did with the technology, we have more information to base our instructional decisions on.</p> <p>For example, it seems like they overlooked something important in their final application that they had interpreted correctly earlier in the task. So you might want to ask some assessing questions to figure out if you are correct about this assumption and to see if, when working a little more slowly, they make a connection between h in the function equation and the graph.</p> </div> <div style="width: 50%;"> <pre> graph TD A[Attend to students' spoken and written mathematical thinking] <--> B[Attend to students' engagement with the technology] A <--> C[Interpret students' understanding from their spoken and written mathematical thinking] B <--> D[Interpret students' understanding from their engagement with the technology] C <--> D C --> E[Decide how to respond] D --> E </pre> </div> </div> </div>



13 Discuss the tips with the teachers. Answer any questions they may have.

Tips for effective attending and interpreting

1. Before noticing, identify the task's mathematical goals and anticipate how students might respond to the task at hand.
2. When attending, take a moment to get the story straight. Consider students' actions (with and without the technology) and words so that you can construct a mathematical / technological play by play of what occurred.
3. When interpreting, stay connected to the mathematical / technological evidence.



14 The purpose of this slide is to introduce teachers to the language of The 5 Practices for Productive Discussions. (Connect to the reading if they did it prior to this lesson. If not assign it after this for more detail.)

5 Practices for Productive Discussions

Practices that take place while planning for instruction		Practice 0: Setting goals and selecting tasks Specifying learning goals and choosing a high-level task that aligns with those goals.
		Practice 1: Anticipating student responses Exploring how you expect students to solve the task and preparing questions to ask them about their thinking.
Practices that take place during instruction, but are considered while planning	Students work individually or in small groups	Practice 2: Monitoring student work Looking closely as students work on the task and asking questions to assess their understanding and move their thinking forward.
	As you move from small group work to whole class discussion	Practice 3: Selecting student solutions Choosing solutions for students to share that highlight key mathematical ideas that will help you achieve lesson goals.
		Practice 4: Sequencing student solutions Determining the order in which to share solutions to create a coherent storyline for the lesson.
	Whole class discussion	Practice 5: Connecting student solutions Identifying connections among student solutions and to the goals of the lesson that you want to bring out during discussion.

Smith, Steele, & Sherin (2020, p.4)





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The purpose of this slide is to show the connection between the skills of Professional Noticing of Students' Mathematical Thinking and those involved with Orchestrating Productive Discussions. This is to foreshadow the skills they will be working on in the PTMT-ESP modules and the connections among them.

Noticing and the 5 Practices for Productive Discussions

5 Practices	Attend	Interpret	Decide
Anticipate	Teachers <i>anticipate</i> by adopting a students' perspective and considering the strategies, questions, and difficult points that may arise as students complete the chosen task. This includes anticipating the ways you expect students to engage with the technology, where their eyes might be drawn, what representations they might favor, and how they will make sense of what they see as a result of their actions with the technology.		
Monitor	Teachers <i>monitor</i> the work of students in real-time by giving attention to and interpreting their written work and their technology engagement (what they do and what they see as a result of what they do) in a coordinated way. These interpretations provide deeper insight into students' thinking. In addition, some technology platforms allow teachers to monitor all students' work in one place.		
Select	Teachers <i>select</i> particular student examples and technological work that will contribute to a broad developmental understanding of the chosen mathematical goal or concept. These selections are based on the manifest features of student strategies to which teachers have attended and interpreted.		
Sequence	Teachers <i>sequence</i> the selected works by drawing on their TPACK and knowledge of mathematical development to construct a purposeful order for student sharing.		
Connect	Teachers <i>connect</i> students' strategies through discussion. Teachers seek to identify and explore connections between different selected examples as well as encouraging connections to the larger goal of the lesson. Skillful orchestration of connecting discussions once again draws on a teacher's capacity to attend to and interpret contours of fluid discussions and make decisions that enhance the connections among the presented mathematics. When students' work is created using dynamic and/or connected representations it is important that this discussion includes the examination of these representations in dynamic ways to support students' connection making.		

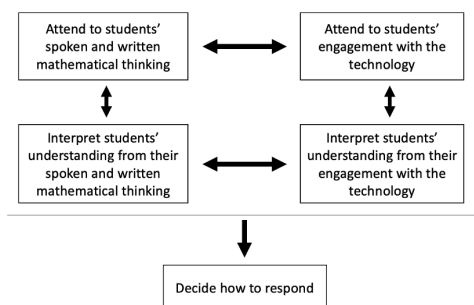
Adapted from Thomas, Fisher, Jong, Schack, Krouse, & Kasten (2015)



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This final slide is a reminder of the framework. Point out that when they are asked to attend to or interpret student thinking with technology they should think about this framework (maybe even pull it out) and make sure they are paying attention to what students say, write, and the ways they engage with the technology (i.e., what they do and what they see when they do it).

Noticing Students' Mathematical Thinking in Technology-Mediated Environments



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Closing Slide



Finally, we recommend as a follow up to this discussion having teachers read and reflect on



[Dick et al. \(2021\) When students use technology tools, what are you noticing?](#)

This article includes additional examples of using the NITE framework to notice student thinking in technology-mediated learning environments. The examples cut across age groups, content strands, and use a variety of technology tools. Direct links to each of the videos in the article are included here.



[Video 1](#)



[Video 2](#)



[Video 3](#)



[Video 4](#)